HIROSHIMA MATH. J.
7 (1977), 657–665

Forced Nonoscillations in Second Order Functional Equations

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(Received June 8, 1976)

1. Introduction

Recently a great deal of effort has been spent in obtaining criteria and asymp-ptotic properties of oscillatory and nonoscillatory solutions of equations of the type

$$(r(t)y'(t))' + p(t)f(y(t)) = q(t). \tag{1}$$

The reader is referred to the works of Graef and Spikes [3, 4], Kusano and Onose [6], this author [9, 10, 11, 12], Skidmore and Bowers [13] and Skidmore and Leighton [14]. Most of these authors assume the nonnegative nature of $p(t)$ to arrive at various oscillation criteria.

Very little seems to be known for retarded equations of the form

$$(r(t)y'(t))' + p(t)f(y(g(t))) = q(t). \tag{2}$$

Standard techniques that have been discovered for equation (1) simply do not work for equation (2). Our purpose in this paper is to study equation (2) and find conditions to force all solutions of equation (2) to be nonoscillatory. We shall first prove that under very general conditions, all solutions of (2) may be continued indefinitely on some positive half real line.

2. Definitions and assumptions

In what follows we shall assume: $r, p, q, g \in C[[t_0, \infty), R]$, $t_0 > 0, f \in C[R, R]$, $xf(x) > 0$, $f(x)/x \leq m$, $m > 0$, $r(t) > 0$, $g(t) \leq t$, $g'(t) > 0$, $g(t) \to \infty$ as $t \to \infty$. We call a function $h \in C[[t_0, \infty), R]$ to be oscillatory if $h(t)$ has arbitrarily large zeros in $[t_0, \infty)$; otherwise we call $h(t)$ nonoscillatory.

3. Indefinite continuation of solutions

In this section, we prove that under very general conditions, all solutions of equation (2) can be continued indefinitely to the right of $t_0 > 0$. See [2, Theorem (2.1)].