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On p-Indicators in Ext(Q/Z, T)

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§1. Introduction

All groups considered in this paper are abelian groups and written additively. We mention basic notations and terminology here. Further details may be found in [1] and [2].

A group A is *divisible* if $nA=A$ for every integer n ($\neq 0$). A is *reduced* if A contains no divisible subgroups ($\neq\{0\}$). A is said to be *p-divisible* if $pA=A$ for some prime p. Let p be a prime and σ be an ordinal. If $\sigma-1$ exists, $p^{\sigma}A=p(p^{\sigma-1}A)$; if σ is a limit ordinal, $p^{\sigma}A=\cap p^{\rho}A$ ($\rho<\sigma$). The σ-th *Ulm subgroup*, denoted by A^{σ}, is $\cap_{p}p^{\omega\sigma}A$, where p runs over all primes.

There is a least ordinal λ such that $p^{\lambda}A$ is p-divisible. λ is called the *p-length* of A. If x is an element of A, $h_p(x)$ shall denote the *p-height* of x in A as follows: if $x\in p^{\sigma}A\backslash p^{\sigma+1}A$, $h_p(x)=\sigma$; if $x\in p^{\sigma}A=p^{\sigma+1}A$ for some σ, $h_p(x)=\infty$ where ∞ is considered to be larger than every occurring ordinal. Set $h_p(p^n x)=\sigma_n$ for $n=0, 1, 2,\cdots$. We call the sequence of ordinals and ∞'s (σ_0, σ_1, $\sigma_2,\cdots$) the *p-indicator* of x. If $\sigma_n+1<\sigma_{n+1}$, then the p-indicator of x is said to have a *gap* between σ_n and σ_{n+1}. Let $p_1,\cdots, p_n,\cdots$ be the sequence of all primes. With a given element x, we associate the *height matrix*

$$\begin{pmatrix} \sigma_{10}\sigma_{11}\cdots \\ \cdots\cdots\cdots\cdots \\ \sigma_{n0}\sigma_{n1}\cdots \\ \cdots\cdots\cdots\cdots \end{pmatrix}$$

whose n-th row is the p_n-indicator of x.

A subgroup G of A is called *pure*, if $nG=G\cap nA$ holds for every integer n. G is called *isotype*, if $p^{\sigma}G=G\cap p^{\sigma}A$ for all ordinals σ and primes p. If this relation holds for some prime p, G is said to be *p-isotype* in A.

If a group A contains both nonzero elements of finite order and elements of infinite order, A is called *mixed*. The *torsion-free rank* of a group A is the cardinality of an independent subset of A which contains only elements of infinite order and which is maximal with respect to this property.

A group A is called *cotorsion* if every extension of A by a torsion-free group splits. A cotorsion group that is reduced and has no nonzero torsion-free direct summands is called *adjusted*. A group A is called *algebraically compact* if A is a direct summand in every group G that contains A as a pure subgroup.