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Some General Properties of Behavior Spaces of Harmonic Semiexact Differentials on an Open Riemann Surface

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Introduction

Beside the original definition due to Kusunoki [3], there are several different ways to define semiexact canonical differentials (see Kusunoki [4, 5], Mori [9]; cf. also Ahlfors-Sario [2], Mizumoto [8] and Yoshida [13]). Above all, the following characterization of semiexact canonical differentials also by Kusunoki ([4]) is remarkable: Let R be an open Riemann surface and φ a meromorphic semiexact differential on R. Then φ is a semiexact canonical differential if and only if there is a canonical region R' on R such that (i) the real part du of φ is exact and square integrable on $R-R'$, and (ii) for any square integrable real harmonic semiexact differential ω on $R-R'$ the mixed Dirichlet integral $(du, \omega^*)_{R-R'}$ of du and ω^* over $R-R'$ is equal to the contour integral $\int_{\partial R'} u\omega$.

A similar characterization is obtained for harmonic differentials with Γ_χ-behavior in the sense of Yoshida ([13], in particular, pp. 186–187). Since, as is well known (cf. [5], [9]), semiexact canonical differentials correspond to one of the special extreme cases, the case $\Gamma_\chi = \Gamma_{hm}$ (the space of real harmonic measures on R), the results in [13] is certainly a generalization of Kusunoki's characterization. On the other hand, we considered in [11] spaces of (complex) harmonic semiexact differentials with certain simple properties and called them behavior spaces. We also showed that we can use such a behavior space Λ_0 to describe a more general boundary behavior, Λ_0-behavior, of analytic (meromorphic) differentials.

The aim of the present article is to show some properties of behavior spaces. It is easy to see that we can apply the very same definition of Λ_0-behavior not only to analytic differentials but also to C^1-differentials (defined near the ideal boundary of R). See Definition 3. We shall generalize some of Kusunoki's characterizations of semiexact canonical differentials to the case of C^1-differentials with Λ_0-behavior. Then we shall introduce an equivalence relation among behavior spaces on R. We can easily see that Λ_0- and $\tilde{\Lambda}_0$-behaviors are the same if and only if Λ_0 is equivalent to $\tilde{\Lambda}_0$. In other words, Λ_0-behavior is determined by the equivalence class of Λ_0. As an immediate consequence of this, we know that