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On a p-Capacity of a Condenser and KD^p-Null Sets

Hiromichi YAMAMOTO

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Introduction

Ahlfors and Beurling [1] introduced the notion of null sets of class N_D in the complex plane and characterized such null sets by means of the extremal length. Hedberg [6] considered a generalization of this notion, namely, removable sets for the class FD^p $(1<p<\infty)$ in an N-dimensional euclidean space R^N, and characterized such removable sets by means of condenser capacities. We can consider a class KD^p of p-precise functions on R^N $(N\geqq 3)$ and define KD^p-null sets. In the present paper, we shall show several relations between KD^p-null sets and p-capacities of a condenser.

A real valued function u defined in a domain D of R^N is called a p-precise function, if it is absolutely continuous along p-a.e. curve in D and $|\mathrm{grad}\, u|$ belongs to $L^p(D)$. A p-precise function u in D has a finite curvilinear limit $u(\gamma)$ along p-a.e. curve γ in D (see [9, Theorem 5.4]). Let α be a compact subset of ∂D and $\Gamma_D(\alpha)$ be the family of all locally rectifiable curves in D each of which starts from some point of D and tends to α. Let α_0, α_1 be non-empty compact subsets of ∂D such that $\alpha_0\cap\alpha_1=\emptyset$. We follow [9] in defining the p-capacity of condenser $(\alpha_0, \alpha_1; D)$:

$$C_p(\alpha_0, \alpha_1; D) = \inf_u \int_D |\mathrm{grad}\, u|^p dx,$$

where the infimum is taken over all p-precise functions u in D such that $u(\gamma)=0$ (resp. 1) for p-a.e. $\gamma\in\Gamma_D(\alpha_0)$ (resp. $\Gamma_D(\alpha_1)$). Denote by $\hat{D}$ the Kerékjártó-Stoïlow compactification of D. For a condenser $(\alpha_0, \alpha_1; D)$ such that α_0 and α_1 are two mutually disjoint closed subsets of $\hat{D}-D$ and a partition $\{\beta_\iota\}$ of $\hat{D}-D-\alpha_0-\alpha_1$, we shall consider a new kind of p-capacity $C_p^*(\alpha_0, \alpha_1; D, \{\beta_\iota\})$ as follows. Let the boundary components of D be divided into α_0, α_1 and $\{\beta_\iota\}$. We set

$$C_p^*(\alpha_0, \alpha_1; D, \{\beta_\iota\}) = \inf_u \int_D |\mathrm{grad}\, u|^p dx,$$

where the infimum is taken over all p-precise functions u in D such that $u(\gamma)=0$ (resp. 1) for p-a.e. $\gamma\in\Gamma_D(\alpha_0)$ (resp. $\Gamma_D(\alpha_1)$) and $u(\gamma)=a_\iota$ for p-a.e. $\gamma\in\Gamma_D(\beta_\iota)$, where each a_ι is a constant depending on u. On the other hand, we take an