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A Posteriori Error Estimates and Iterative Methods in the Numerical Solution of Systems of Ordinary Differential Equations

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1. Introduction

Consider the boundary value problem

$$\frac{dx}{dt} = X(x, t), \qquad a \leqq t \leqq b, \tag{1.1}$$

$$f[x] = 0, \tag{1.2}$$

and let $x_0(t)$ be an approximate solution of this problem, where x and $X(x, t)$ are real n-vectors, f is an operator from $D \subset C[J]$ into R^n which is continuously Fréchet differentiable in D, and $C[J]$ is the space of all real n-vector functions continuous on $[a, b]$.

Constructing an operator equation from (1.1), (1.2) and approximating the Fréchet derivative of the operator in a neighborhood of x_0 by a linear operator independent of x, by means of an iterative method Urabe [7] proved the existence and local uniqueness of an exact solution and gave an a posteriori error estimate of x_0 in terms of $x_0(t)$ and its derivative.

The first object of this paper is to obtain the results similar to those in [7] for continuous $x_0(t)$ without assuming its differentiability. This is achieved by replacing (1.1) with an equivalent system of integral equations. Hence the results can be applied to discrete numerical solutions by means of interpolation.

The second object of this paper is to treat the case where the linear operator approximating the Fréchet derivative depends on x. This enables us to construct various iterative methods.

In Section 3 the results are applied to multipoint boundary value problems [5, 6]. In Section 4 we consider boundary value problems of the least squares type [1, 8] which arise often in system identification problems and propose some iterative methods.

2. Convergence of iterative methods and error estimates

Let R^n denote a real n-space with any norm $\|\cdot\|$ and let $C[J]$ be the space