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# *A Note on Vector Fields up to Bordism*

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## §1. Introduction

For a (differentiable) closed $m$-manifold $M^m$, let $\operatorname{Span} M^m$ denote the maximum number of linearly independent (tangent) vector fields on $M^m$, and $w_i M^m$ the $i$-th Stiefel-Whitney class of $M^m$. Then by [2, p. 39], we have the following:

(1.1) *If* $\operatorname{Span} M^m \geqq k$, *then* $w_i M^m = 0 \quad (i \geqq m - k + 1)$.

The converse of (1.1) is not true. The purpose of this note is to prove the following

THEOREM. *Let $M^m$ be a closed m-manifold for which all Stiefel-Whitney numbers divisible by $w_m, \cdots, w_{m-k+1}$ are zero. If $k \leqq 6$, then there exists a closed m-manifold $N^m$ such that $N^m$ is unorientedly bordant to $M^m$ and* $\operatorname{Span} N^m \geqq k$.

By R. E. Stong [3, p. 440], the following conjecture is proved for $k=1, 2, 4$: Under the assumption of the theorem, $M^m$ is unorientedly bordant to a manifold $N^m$ which is fibered over the product $(S^1)^k$ of $k$-copies of the circle $S^1$. It is clear that the theorem holds if this conjecture is true.

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## §2. Some manifolds having many vector fields

For a real (differentiable) $n$-plane bundle $\zeta \to V$ over a closed $m$-manifold $V$, we denote by $p: RP(\zeta) \to V$ the associated projective space bundle with fiber $RP(n-1)$ (the real projective $(n-1)$-space). Then $RP(\zeta)$ is a closed $(m+n-1)$-manifold and

(2.1) *the cohomology with $Z_2$ coefficients of $RP(\zeta)$ is the free module over the cohomology of $V$ on $1, c, \cdots, c^{n-1}$, with the relation*

$$c^n = \sum_{i=1}^{n} p^*(w_i\zeta)c^{n-i},$$

*where $c$ is the first Stiefel-Whitney class of the canonical line bundle over $RP(\zeta)$ and $w_i\zeta$ is the i-th Stiefel-Whitney class of $\zeta$.*