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On Nonoscillatory Solutions of Functional Differential Equations with a General Deviating Argument

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1. Introduction

The equation to be studied in this paper is

$$x^{(n)}(t) + \sigma f(t, x(g(t))) = 0, \tag{A}$$

where the following conditions are always assumed to hold:

(a) $n \geqq 2, \sigma = \pm 1$;

(b) $g(t)$ is continuous on $[a, \infty)$ and $\lim_{t\to\infty} g(t) = \infty$;

(c) $f(t, x)$ is continuous on $[a, \infty)\times(-\infty, \infty)$ and $xf(t, x) \geqq 0$.

It is to be noted that $g(t)$ is a general deviating argument, that is, it is allowed to be *retarded* ($g(t) \leqq t$) or *advanced* ($g(t) \geqq t$) or otherwise.

Equation (A) is called *superlinear* if, for each t,

$$|f(t, x_1)|/|x_1| \geqq |f(t, x_2)|/|x_2| \quad \text{for} \quad |x_1| > |x_2|, \ x_1x_2 > 0,$$

and *strongly superlinear* if there is a number $\alpha > 1$ such that, for each t,

$$|f(t, x_1)|/|x_1|^\alpha \geqq |f(t, x_2)|/|x_2|^\alpha \quad \text{for} \quad |x_1| > |x_2|, \ x_1x_2 > 0.$$

Dually, equation (A) is called *sublinear* if, for each t,

$$|f(t, x_1)|/|x_1| \leqq |f(t, x_2)|/|x_2| \quad \text{for} \quad |x_1| > |x_2|, \ x_1x_2 > 0,$$

and *strongly sublinear* if there is a positive number $\beta < 1$ such that, for each t,

$$|f(t, x_1)|/|x_1|^\beta \leqq |f(t, x_2)|/|x_2|^\beta \quad \text{for} \quad |x_1| > |x_2|, \ x_1x_2 > 0.$$

In this paper we are primarily interested in the nonoscillatory solutions of equation (A) which is either strongly superlinear or strongly sublinear. Of particular interest is the effect that $g(t)$ can have on the nonoscillation properties of (A). Hereafter, the term "solution" will be used to mean a solution $x(t)$ of (A) which is defined on some half-line $[T_x, \infty)$ and is nontrivial on any infinite subinterval of $[T_x, \infty)$. Such a solution is said to be *oscillatory* if it has arbitrarily large zeros; otherwise it is said to be *nonoscillatory*.