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*Oscillatory and Asymptotic Behavior of the Bounded Solutions of Differential Equations with Deviating Arguments**)

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1. Introduction

Let r_i $(i=0, 1,\dots, n)$ be positive continuous functions on the interval $[t_0, \infty)$. For a real-valued function h on $[T, \infty)$, $T \geqq t_0$, and any $\mu=0, 1,\dots, n$ we define the *μ-th r-derivative* of h by the formula

$$D_r^{(\mu)}h = r_\mu(r_{\mu-1}(\cdots(r_1(r_0h)')'\cdots)')' .$$

Then we obviously have

$$D_r^{(0)}h = r_0h \quad \text{and} \quad D_r^{(i)}h = r_i(D_r^{(i-1)}h)' \qquad (i = 1, 2,\dots, n).$$

Moreover, if $D_r^{(n)}h$ is defined on the interval $[T, \infty)$, then the function h is said to be *n-times r-differentiable* and if, in addition, $D_r^{(n)}h$ is continuous, h is said to be *n-times continuously r-differentiable.* If $r_i=1$ $(i=0, 1,\dots, n)$, this notion specializes to the one of the usual differentiability.

Now, we consider the n-th order $(n>1)$ differential equation with deviating arguments of the form

$$(E, \delta) \qquad (D_r^{(n)}x)(t) + \delta F(t; x < g(t) >) = b(t),$$

where $r_n=1$, $\delta=\pm 1$ and

$$x<g(t)> = (x[g_1(t)], x[g_2(t)],\dots, x[g_m(t)]), \quad g = (g_1, g_2,\dots, g_m).$$

The continuity of the real-valued functions F on $[t_0, \infty)\times \mathbf{R}^m$ and g_i $(i=1, 2,\dots, m)$, b on $[t_0, \infty)$ as well as sufficient smoothness for the existence of solutions of (E, δ) on an infinite subinterval of $[t_0, \infty)$ will be assumed without mention. In what follows the term "solution" is always used only for such solutions $x(t)$ of (E, δ) which are defined for all large t. The oscillatory character is considered in the usual sense, i.e. a continuous real-valued function which is defined on an interval of the form $[T, \infty)$ is called *oscillatory* if it has no last zero, and otherwise

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