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Oscillation and Nonoscillation for Perturbed Differential Equations

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1. Introduction

Consider the equation

$$\text{(I)} \qquad x^{(n)} + H(t, x) = Q(t, x), \quad n \text{ even},$$

where H, Q are real continuous functions defined on $[0, \infty) \times (-\infty, \infty)$. The following theorem was given by the author in [4]:

THEOREM A. *Let $H(t, u)$ be increasing in u, $uH(t, u) > 0$ for $u \neq 0$ and such that all bounded solutions of*

$$\text{(II)} \qquad x^{(n)} + H(t, x) = 0$$

oscillate. Moreover, let $|Q(t, x)| \leq Q_0(t)|x|^r$, where $r \geq 1$, Q_0: $[0, \infty) \to [0, \infty)$, continuous and such that

$$\int_0^\infty t^{n-1} Q_0(t) dt < +\infty.$$

Then every bounded solution of (I) *oscillates.*

As it was shown in [4], this theorem does not necessarily hold for $r < 1$, or for functions Q_0 with

$$\int_0^\infty t^{n-1} Q_0(t) dt = +\infty,$$

or for all solutions of (I), provided of course that all solutions of (I) oscillate. In this paper we provide conditions under which an nth order functional differential equation of the form

$$\text{(III)} \qquad x^{(n)} + H(t, x(g_1(t))) = Q(t, x(g_2(t)))$$

has all of its bounded solutions oscillatory. In the particular case $g_1(t) \equiv t$, $g_2(t) \equiv t$, this result does not necessarily demand that the perturbation Q be superlinear or small as in Theorem A. Next, we provide some results under which all solutions of (III) with $g_1(t) \equiv g_2(t) \equiv g(t)$ either oscillate, or are such that the