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On the Fixed Points of Elliptic Elements of B-Groups

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Recently there have been many results ([1], [4]) concerning the properties of B-groups, but few informations about the location of the fixed points of their elliptic elements.

In this paper we shall give some properties of fixed points of these elements. Before stating our theorem we shall explain notation.

Let G be a B-group, $\Lambda(G)$ the limit set of G, and $E(G)$ the set of fixed points of elliptic elements in $\Lambda(G)$. Let $E_d(G)$ (resp. $E_e(G)$) be the subset of $E(G)$ consisting of the fixed points of the elements conjugate in G to the elliptic elements in some degenerate (resp. elementary) group in $\{G_1,\dots, G_t\}$ (see section 1).

Our main result is the following theorem.

THEOREM. *Let G be a B-group with a simply connected invariant component Δ_0. Then the following three propositions hold:*

(1) $E(G)=E_d(G) \cup E_e(G)$ *and* $E_d(G) \cap E_e(G)=\emptyset$.

(2) *If* $E_d(G)\neq\emptyset$, *then G is not regular.*

(3) *For any $z \in E_d(G)$, its stability subgroup $G_z=\{E|E(z)=z, E \in G\}$ is an elliptic cyclic group and z can not lie on the boundaries of components except Δ_0.*

1. Let us begin with recalling some notation and definitions.

Let G be a kleinian group. Denote by $\Omega(G)$ the region of discontinuity of G, and $\Omega(G)'$ the set of points z with the property that each z has a neighborhood W such that $V(W) \cap W=\emptyset$ for all $V\in G$, $V\neq 1$. Then $\Omega(G)-\Omega(G)'$ consists of isolated fixed points of elliptic elements of G and the stability group $G_z=\{E|E(z)=z, E\in G\}$ for any $z \in \Omega(G)-\Omega(G)'$ is an elliptic cyclic group.

The components of $\Omega(G)$ are called components of G. A component Δ_0 of G is called invariant if $V(\Delta_0)=\Delta_0$ for every $V\in G$. For each component Δ of G, let G_Δ be the subgroup of G which keeps Δ invariant, and set $\Delta'=\Delta \cap \Omega(G)'$. Then $S=\Delta'/G_\Delta$ is a Riemann surface and the canonical projection $\Delta'\to S$ is conformal. If $\{\Delta_0, \Delta_1,\dots\}$ is a complete list of non-conjugate components of G and if $S_i=\Delta_i'/G_{\Delta_i}$, then $\Omega(G)'/G=S_0+S_1+\cdots$. The surfaces S_0, $S_1,\dots$ are called the factors of G.

A finitely generated, non-elementary kleinian group G with a simply con-