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Classification Theory for Nonlinear Functional-Harmonic Spaces

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Introduction

In the classical classification theory of Riemann surfaces, the basic relations involving classes of harmonic functions are given by

$$O_G \subsetneqq O_{HP} \subsetneqq O_{HB} \subsetneqq O_{HD} = O_{HDB} \tag{1}$$

(see, e.g., [11] for notation and detailed account of the classical classification theory). The same relations have been shown to hold for the class H of solutions of the equation of the form

$$\Delta u = Pu \qquad (P \geqq 0) \tag{2}$$

on, in general, Riemannian manifolds Ω; furthermore, for the solutions of (2), additional relations

$$O_{HD} \subsetneqq O_{HE} = O_{HBE} \tag{3}$$

hold, where E indicates the finiteness of the energy integral

$$\int_\Omega |\nabla u|^2 dx + \int_\Omega Pu^2 dx \qquad (dx\text{: the volume element}) \tag{4}$$

(see, e.g., [9], [5]).

Here, we note that (2) is the Euler equation of the variational integral (4). Thus we may generalize the above situation as follows. For simplicity, consider the case where Ω is a domain in the euclidean space $\mathbf{R}^d$. Suppose the "Dirichlet integral" of a function f is given in the form

$$D[f] = \int_\Omega \psi(x, \nabla f(x)) dx \tag{5}$$

with a function $\psi(x, \tau)$: $\Omega \times \mathbf{R}^d \to \mathbf{R}$ which is non-negative and convex in τ, and the "energy" of f is given by

$$E[f] = D[f] + \int_\Omega \Gamma(x, f(x)) dx \tag{6}$$