HIROSHIMA MATH. J.
8 (1978), 223–233

Some Boundary Value Problems for the Hamilton-Jacobi Equation

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(Received August 15, 1977)

1. Introduction

This paper is devoted to the study of some initial value-boundary value problems for the Hamilton-Jacobi equation

$$\frac{\partial u}{\partial t} + F\left(\frac{\partial u}{\partial x}\right) = 0 \qquad (t \geq 0,\ x \geq 0)\,. \tag{1}$$

It will be shown that (1) is governed by contraction semigroups on certain closed subsets of the space of bounded uniformly continuous functions on the half-line $\mathbf{R}^+ = [0, \infty)$ via the Crandall-Liggett generation theorem [10] for nonlinear semigroups.

Preparatory to these results, results concerning existence of periodic solutions, positive solutions, and even solutions for the Cauchy problem for (1) with $x \in \mathbf{R} = (-\infty, \infty)$ will be obtained.

The Cauchy problem for the Hamilton-Jacobi equation has been studied from a semigroup point by Aizawa [2, 3], Burch [7], and Tamburro [15, 16]. The results and techniques in [7] are refined and further developed in this paper to gain information about some boundary value problems for (1). Results concerning these problems complement the recent work of Feltus [11]. Feltus studied existence and uniqueness for (1) with Dirichlet boundary condition at the origin, but he didn't establish continuous dependence results. Some earlier results on boundary value problems for (1) were obtained by Aizawa and Kikuchi [1, 4] and Benton [5, 6].

Before stating the main result we introduce some notation. J denotes either $\mathbf{R} = (-\infty, \infty)$ or $\mathbf{R}^+ = [0, \infty)$. For $1 \leq p \leq \infty$, $L^p(J)$ denotes the usual real Lebesgue space with norm $\|\cdot\|_p$. $W^{n,p}(J)$ denotes the Sobolev space of all $f \in L^p(J)$ such that the jth derivative of f belongs to $L^p(J)$ for $j \leq n$. BUC(J) denotes the bounded uniformly continuous real functions on J. For $Y(\mathbf{R})$ any space of functions on $\mathbf{R}$ we *define*

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* Partially supported by NSF grant MCS 75–21416.
† Partially supported by NSF grant MCS 76–06515.