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# *On a Mixed Problem for the Multi-Dimensional Hamilton-Jacobi Equation in a Cylindrical Domain*

Yoshihito TOMITA

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## 1. Introduction

Let $\Omega$ be a bounded domain in $\boldsymbol{R}^n$ with smooth boundary $\partial\Omega$, and $Q$ be the cylinder $(0, \infty) \times \Omega$. We consider the mixed initial and boundary value problem (hereafter called (MP)) for the Hamilton-Jacobi equation in $Q$:

$$u_t + H(t, x, u, u_x) = 0, \qquad (t, x) \in Q, \tag{1.1}$$

$$u(0, x) = u_0(x), \qquad x \in \bar{\Omega}, \tag{1.2}$$

$$u(t, x) = \phi(x), \qquad (t, x) \in \boldsymbol{R}^+ \times \partial\Omega. \tag{1.3}$$

Here $\bar{\Omega}$ and $\boldsymbol{R}^+$ denote $\bar{\Omega} = \Omega \cup \partial\Omega$ and $\boldsymbol{R}^+ = [0, \infty)$ respectively, $u(t, x)$ is a real-valued function, $H$: $\boldsymbol{R}^+ \times \bar{\Omega} \times \boldsymbol{R}^1 \times \boldsymbol{R}^n \to \boldsymbol{R}^1$, and $u_x$ denotes the gradient $(u_{x_1}, \dots, u_{x_n})$ in the space variables $x$.

The purpose of this paper is to establish the existence and uniqueness of global generalized solutions of (MP). We employ the so-called vanishing viscosity method in proving existence for (MP). The reason for the employment of this method lies in its advantage in estimating the local semi-concavity constant which will be described in the next section. As an intermediate step in the development, we shall solve a mixed problem for a nonlinear second-order parabolic equation by making use of the semigroup approximation theory. The semigroup approach enables us not only to prove the existence of a (generalized) solution of the mixed problem for regularized parabolic equations, but also to employ the vanishing viscosity method.

This investigation is a sequel to our earlier work [20] and is motivated by the works of Aizawa [1, 3] and Kružkov [15]. Aizawa [1] treated the Cauchy problem for the Hamilton-Jacobi equation in one space variable

$$u_t + f(u_x) = 0, \qquad t > 0, \quad -\infty < x < +\infty, \tag{$*$}$$

from the viewpoint of the nonlinear semigroup theory, and constructed a global generalized solution, assuming only that $f$ is continuous. He subsequently studied the Cauchy problem for the multi-dimensional equation of this type from