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Regular Rings, V-Rings and their Generalizations

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In this paper, the notions of right $p.p.$ rings, right CPP-rings and right CPF-rings, introduced primarily for rings with identity, will be defined for s-unital rings. One of the purposes of this paper is to extend the principal results in [19] to s-unital rings so as to improve several previous results obtained in [15–18] (Theorems 1–5). Furthermore, we shall present a characterization of an s-unital right CPP-ring (Theorem 6), which will deduce the main theorem in [6].

Throughout A will represent a ring (possibly without identity). Given a right (resp. left) ideal I of A, I^* will denote the intersection of all maximal right (resp. left) ideals of A containing I. If M is a right (resp. left) A-module and S is a subset of A, then we set $\ell_M(S)=\{u\in M|uS=0\}$ (resp. $r_M(S)=\{u\in M|Su=0\}$). As usual, we write $\ell(S)=\ell_A(S)$ and $r(S)=r_A(S)$. As for other notations and terminologies used in this paper, we follow the previous ones [15] and [16].

1. Preliminaries. Following [15], a non-zero right (resp. left) A-module M is said to be *s-unital* if $u\in uA$ (resp. $u\in Au$) for each $u\in M$. If A_A (resp. ${}_AA$) is s-unital, A is called a *right* (resp. *left*) *s-unital ring*. In case A is right and left s-unital, we merely say s-unital. We begin by stating a lemma which will be used repeatedly in what follows.

LEMMA 1 ([15, Theorem 1] and [11, Lemma 1 (a)]). *If F is a finite subset of a right s-unital ring (resp. an s-unital ring) A, then there exists an element $e\in A$ such that $ae=a$ (resp. $ea=ae=a$) for all $a\in F$.*

A right A-module M is said to be *p-injective* if for any principal right ideal $|a)$ of A and $f:|a)_A\to M_A$ there exists an element $u\in M$ such that $f(x)=ux$ for all $x\in|a)$. As is well known, A is a regular ring if and only if every right A-module is p-injective.

LEMMA 2 (cf. [4, Proposition 1.7 and Corollary 1.9]). *Let A be a right s-unital ring, and M_A an s-unital module. If M_A is p-injective then, for each $a\in A$, there holds $\ell_M(r(a))=Ma$, and conversely. In particular, for a domain A with 1, a unital module M_A is p-injective if and only if M_A is divisible.*

PROOF. Assume that M_A is p-injective. Given $u\in\ell_M(r(a))$, there exists