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S^1-Actions on Cohomology Complex Projective Spaces with Three Components of the Fixed Point Sets

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§0. Introduction

A closed oriented smooth $2n$-manifold M is called a *cohomology complex projective n-space* (*cohomology* CP^n), if the integral cohomology ring of M is isomiphic to that of the complex projective n-space CP^n, i.e., if there exists an element $\alpha \in H^2(M; Z)$ such that

$$H^*(M; Z) = Z[\alpha]/(\alpha^{n+1}), \quad <\alpha^n, [M]> = 1. \tag{0.1}$$

Then the conjecture of T. Petrie [4; Intro.] for homotopy complex projective spaces follows immediately from the following statement:

(0.2) *Assume that M is a cohomology CP^n with $\alpha \in H^2(M; Z)$ satisfying* (0.1). *If M admits a non-trivial (smooth) S^1-action, then the total Pontrjagin class of M is given by*

$$p(M) = (1 + \alpha^2)^{n+1}.$$

K. Wang [6] and T. Yoshida [7] have proved independently the conjecture of T. Petrie for semi-free actions by the following

THEOREM ([6; Prop. 2.2–3, Cor. 2.5]). (0.2) *is valid, if M admits an S^1-action whose fixed point set has two (connected) components.*

The purpose of this note is to prove the following

THEOREM 1. (0.2) *and the conjecture of T. Petrie are valid, if M admits an S^1-action whose fixed point set has three components.*

We shall prove this theorem by the following

THEOREM 2. *Let M be a cohomology CP^n. Then any effective S^1-action on M with three components of the fixed point set is of the linear type.*

Here, an S^1-action on M is defined to be of the *linear type*, if its normal representations of the fixed point set are of the same type as those of a linear S^1-action on CP^n (cf. Definition 1.6).