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On the Jacobson Radicals of Infinite Dimensional Lie Algebras

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1.

Recently the infinite-dimensional Lie algebras have been investigated by several mathematicians. The purpose of this paper is to study the Jacobson radicals of infinite-dimensional Lie algebras.

We employ the notation and terminology in [1].

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We denote by L a not necessarily finite-dimensional Lie algebra over a field $\mathfrak{k}$ throughout the paper.

DEFINITION. The Jacobson radical of L is defined to be the intersection of all the maximal ideals of L, with the convention that this intersection is L if there are no maximal ideals. We denote the Jacobson radical of L by J_L.

LEMMA 1. $$J_L \subseteqq L^2.$$

PROOF. Let $L \neq L^2$. If $x \notin L^2$, take a subspace M of L which is complementary to $<x>$ and contains L^2. Then M is a maximal ideal of L. Hence $x \notin J_L$. Therefore $J_L \subseteqq L^2$.

We introduce the following class $\mathfrak{R}$: A Lie algebra L belongs to the class $\mathfrak{R}$ if and only if

$$[L/N, L/N] \subsetneqq L/N$$

for every proper ideal N of L.

It is clear that $\mathrm{E}\mathfrak{A} \subseteqq \mathfrak{R}$.

LEMMA 2. *For an arbitrary ideal K of L belonging to $\mathfrak{R}$, $J_L \supseteqq L^2 \cap K$.*

PROOF. We may assume that $K \neq 0$. Suppose that there is a maximal ideal I of L such that

$$I \not\supseteqq L^2 \cap K.$$

Then $I \not\supseteqq L^2$ and $I \not\supseteqq K$. Thus $L = I + K$. We see that