HIROSHIMA MATH. J.
9 (1979), 7–16

Estimates on the Support of Solutions of Elliptic Variational Inequalities in Bounded Domains

Naoki YAMADA

(Received May 29, 1978)

1. Introduction

To the general question proposed in J. L. Lions [6] as to when the solution of a variational inequality has a compact support, H. Brezis [4], A. Bensoussan and J. L. Lions [1], H. Brezis and A. Friedman [5] have given various affirmative answers for solutions of stationary or evolutionary variational inequalities.

In the present note we shall consider the solution u of an elliptic (stationary) variational inequality of the form

$$\text{(VI)} \qquad \begin{aligned} &-\Delta u+\alpha u \geqq f, \qquad u \geqq \Psi, \\ &(u-\Psi)(-\Delta u+\alpha u-f)=0 \qquad \text{in } \Omega \end{aligned}$$

under various boundary conditions, where Ω is a bounded domain in R^N, Δ denotes the Laplace operator, and α is a positive constant.

By a solution u of (VI), the domain Ω is divided into two subdomains Ω_1 and Ω_2 such that

$$\begin{aligned} &\Omega_1=\{x \mid u=\Psi\} && \text{(coincidence set)}, \\ &\Omega_2=\{x \mid -\Delta u+\alpha u=f\} && \text{(continuation set)}. \end{aligned}$$

Recently, A. Bensoussan, H. Brezis and A. Friedman [2] obtained an estimate on the size of Ω_1 under the Dirichlet boundary condition.

The purpose of the present note is to obtain some estimates on the size of Ω_1 under other boundary conditions (Neumann, mixed and Signorini). Our main results in this note are stated in section 3 (Theorems 3.2, 3.3 and 3.4). Section 4 is devoted to the study of the behavior of solutions of (VI) near the boundary of Ω. It seems interesting to the author that estimates of the same type can be derived for these different boundary conditions by computing only one comparison function.

2. A comparison theorem

Let Ω be a bounded domain in R^N with smooth boundary Γ. For a maximal