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Structure of Rings Satisfying (Hm) and (Ham)

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All rings considered in this paper are commutative but may not have a unity. An ideal A of a ring R is said to be a multiplication ideal if for every ideal B of R, $B \subseteq A$, there is an ideal C of R such that $B = AC$. An ideal A is said to be an M-ideal if for every ideal B containing A, there is an ideal C such that $A = BC$. R is said to be a multiplication ring if every ideal of R is a multiplication ideal (equivalently every ideal is an M-ideal). A ring R is said to be an (AM)-ring if for any two ideals A and B of R, $A < B$, there is an ideal C of R such that $A = BC$. An ideal A is said to be simple if there is no ideal A' with $A^2 < A' < A$. A ring R is said to be primary if R has at most one proper prime ideal. R is said to be a special primary ring if R has a prime ideal P such that every ideal of R is a power of P. If S is a multiplicatively closed subset of R and A is any ideal then A^e denotes the extension of A to the quotient ring R_S and A^{ec} denotes the contraction of A^e to R. A ring is said to satisfy $(*)$-condition if every ideal with prime radical is primary. A ring R is said to satisfy (Hm) or (Ham) according as every proper homomorphic image of R is a multiplication ring or an (AM)-ring. The purpose of this note is to determine the structure of rings satisfying (Hm) and (Ham) and the desired structure is given by Theorems 1.7 and 2.5.

1. Let R be a ring and N be its set of nilpotent elements. For any subset S of R, define $S^{\perp} = (N: S) =$ set of all x in R such that $xS \subseteq N$[7, p. 434]. The following lemma is due to Griffin [7, Lemma 7].

LEMMA 1.1. *If for any element x of a ring R there exists an ideal D such that $(x) = D(N + (x) + x^{\perp})$ then there is an idempotent $e \in (x^{\perp})^{\perp}$ and a positive integer n such that $x^n = ex^n$.*

LEMMA 1.2. *If R is a ring satisfying (Hm) and $x \in R$ such that $x^2 \neq 0$ then (x) is an M-ideal.*

PROOF. Suppose A is any ideal of R such that $x \in A$. Now $(x)/(x^2) \subseteq A/(x^2)$ in $R/(x^2)$ which is a multiplication ring. There is an ideal I containing x^2 such that $(x)/(x^2) = (A/(x^2))(I/(x^2))$. Thus $(x) = AI + (x^2) = A(I + (x)) + (x^2) = A(I + (x))$, since $x^2 \in A(I + (x))$. Therefore (x) is an M-ideal.

COROLLARY 1.3. *If R is a ring satisfying (Hm) such that* $\mathrm{rad}(0) = (0)$ *then R is a multiplication ring.*