Claw-Decomposition and Evenly-Partite-Claw-Decomposition of Complete Multi-Partite Graphs

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1. Introduction

The decomposition problems of a graph arise in the filing theory and the combinatorial theory of design of experiments. These problems have been developed by Bermond and Schönheim [2], Bermond and Sotteau [3], Erdös, Sauer and Schaer [5], Huang and Rosa [7] and so on. Yamamoto, Ikeda, Shige-eda, Ushio and Hamada [16] have completely solved the problem of claw-decomposability of a complete graph. The claw-decomposition of a complete graph yields an optimal binary-valued balanced file organization scheme of order two which is called a Hiroshima University balanced file organization scheme of order two (HUBFS$_2$) [17]. A binary-valued balanced file organization scheme is said to be optimal if it has the least redundancy among all possible binary-valued balanced file organization schemes having the same parameters, provided the distribution of records is invariant under the permutation of attributes. A necessary condition and some sufficient conditions for complete graph to be decomposed into a union of subgraphs have also been given by Yamamoto and Tazawa [19]. The subgraph is a generalized graph of a claw which is called a hyperclaw. This hyperclaw decomposition provides us an optimal binary-valued balanced file organization scheme of general order k, which is called an HUBFS$_k$, in the above-mentioned sense [20].

Recently, the decomposition problems of other graphs than a complete graph have been investigated by many authors. Myers [9] has investigated the decomposition problems of the product of a complete graph with itself. Sumner [12] has given some theorems on the 1-factorization. Bermond [1], Schönheim [11] and Wilson [15] have investigated the decomposition problems of the directed complete graphs. The decomposition problems of a complete multi-partite graph have been developed by Cockayne and Hartnell [4], Tazawa, Ushio and Yamamoto [13], Ushio, Tazawa and Yamamoto [14] and Yamamoto, Ikeda, Shige-eda, Ushio and Hamada [16]. Yamamoto, Ikeda, Shige-eda, Ushio and Hamada [16] have completely solved the problem of claw-decomposability of a complete bipartite graph. Ushio, Tazawa and Yamamoto [14] have given a necessary and sufficient condition for a complete m-partite graph $K_m(n, n,..., n)$ with m sets of n points each to be decomposed into a union of