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Sufficient Conditions for Duality Theorems in Infinite Linear Programming Problems

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§1. Introduction with problem setting

There are many sufficient conditions for duality theorems in infinite linear programming problems. In this paper, we shall investigate sufficient conditions for a general duality theorem due to K. S. Kretschmer by the aid of the closedness of the sum of two convex cones and find some relations among well-known sufficient conditions in [3]–[11].

More precisely, let X and Y be real linear spaces which are in duality with respect to the bilinear functional $((\ ,\ ))_1$ and let Z and W be real linear spaces which are in duality with respect to the bilinear functional $((\ ,\ ))_2$. Throughout this paper, we always assume that each space of the paired spaces is assigned the weak topology which is compatible with the duality, so that every topological notion is used without any specifying adjective unless otherwise stated. Let A be a continuous linear transformation from X into Z, P and Q be closed convex cones in X and Z respectively and $y_0 \in Y$ and $z_0 \in Z$ be fixed elements. Denote by A^* the adjoint of A and by P^+ and Q^+ the dual cones of P and Q respectively. Let us consider the following infinite linear programming problem (1.1) and its dual problem (1.2):

(1.1) Find $M = \inf\{((x, y_0))_1;\ x \in S\}$,

where $S = \{x \in P;\ Ax - z_0 \in Q\}$.

(1.2) Find $M^* = \sup\{((z_0, w))_2;\ w \in S^*\}$,

where $S^* = \{w \in Q^+;\ y_0 - A^*w \in P^+\}$.
Here we use the convention that the infimum of a real function on the empty set ϕ is equal to ∞. We say that problem (1.1) has an optimal solution if there exists an $x \in S$ such that $M = ((x, y_0))_1$. A result which assures the equality $M = M^*$ is called a duality theorem.

Let R be the set of real numbers and R^+ be the set of non-negative real numbers. When R is considered as a topological space, the topology is the usual one. Product spaces $Z \times R$ and $W \times R$ are in duality with respect to the bilinear functional $[\ ,\ ]$ defined by $[(z, r), (w, s)] = ((z, w))_2 + rs$ for every $(z, r) \in Z \times R$ and $(w, s) \in W \times R$.