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Isometry Groups of Negatively Pinched 3-Manifolds

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§1. Introduction

Let M be a Riemannian manifold. For a 2-plane π tangent to M, let $k(\pi)$ denote the sectional curvature at π. M is said to be *negatively pinched* if there exist negative numbers c_1 and c_2 such that $c_1 \leq k(\pi) \leq c_2 < 0$ for all π.

In [8], p. 152, Margulis gave some unsolved problems. Among them we find:

PROBLEM. Let M be a simply connected, symmetric space of noncompact type. Let Γ_1 and Γ_2 be discrete groups of isometries of M with the factor spaces of finite volume. Is the ratio of the volume $\mathrm{vol}(M/\Gamma_1)$: $\mathrm{vol}(M/\Gamma_2)$ rational?

The main purpose of this paper is to establish the following theorem by which in particular the analogous problem to the above for negatively pinched 3-manifolds can be reduced to the case of hyperbolic spaces.

THEOREM. *Let M be a complete, simply connected, negatively pinched Riemannian manifold of dimension three. Suppose that there exists a discrete group Γ of isometries of M such that the factor space M/Γ is of finite volume. Then either M is of constant curvature or the group $I(M)$ of all isometries of M is discrete.*

Under the assumptions of the theorem, let G denote the identity component of the Lie group $I(M)$. Then by Heintze, G is a semisimple Lie group without compact factor and with trivial center, see §2, Lemma 6. Now suppose that $G \neq \{1\}$. Then we can see easily that G must be isomorphic with the adjoint group of either $SL(2, \mathbf{C})$ or $SL(2, \mathbf{R})$, and if we can exclude the latter case, the theorem follows directly. For the purpose we start with a little more general setting and construct a warped product of a certain symmetric space and a straight line $\mathbf{R}$, which does not admit a discrete group Γ of isometries with the factor space of finite volume. The exact statement is given in PROPOSITION of §3.

§2. Lemmas

For the later use we shall state some known results.

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