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On Full Uniform Simplification of Even Order Linear Differential Equations with a Parameter

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1. Introduction

We shall be concerned with the system of linear ordinary differential equations with a parameter

$$\varepsilon \frac{dX}{dt} = A(t, \varepsilon)X, \tag{1.1}$$

where the n by n matrix function $A(t, \varepsilon)$ is holomorphic in the domain

$$D(t_0, \varepsilon_0, \theta_0) = \{(t, \varepsilon) \mid |t| \leqq t_0, \quad 0 < |\varepsilon| \leqq \varepsilon_0, \ |\arg \varepsilon| \leqq \theta_0\}$$

and admits the uniform asymptotic expansion

$$A(t, \varepsilon) \sim \sum_{i=0}^{\infty} A_i(t)\varepsilon^i \qquad \text{as} \quad \varepsilon \longrightarrow 0 \quad \text{in} \quad |\arg \varepsilon| \leqq \theta_0. \tag{1.2}$$

The coefficients $A_i(t)$ $(i=0, 1,\dots)$ of (1.2) are holomorphic in the closed disk $|t| \leqq t_0$. We here assume that

$$A_0(t) = \begin{pmatrix} 0 & 1 & & & 0 \\ & 0 & 1 & & \\ & & \ddots & \ddots & \\ & & & \ddots & 1 \\ t^q & 0 & \cdots & \cdots & 0 \end{pmatrix}$$

q being a positive integer, which implies that the origin $t=0$ is a turning point of (1.1).

In order to investigate asymptotic behaviors of solutions of (1.1) in a full neighborhood of the turning point $t=0$, we usually try to find a matrix $Q(t, \varepsilon)$, which is holomorphic in $D(t_1, \varepsilon_1, \theta_1,)$ $(0 < t_1 \leqq t_0,\ 0 < \varepsilon_1 \leqq \varepsilon_0,\ 0 < \theta_1 \leqq \theta_0)$ and admits an asymptotic expansion of the form

$$Q(t, \varepsilon) \sim \sum_{i=0}^{\infty} P_i(t)\varepsilon^i \qquad \text{as} \quad \varepsilon \longrightarrow 0 \quad \text{in} \quad |\arg \varepsilon| \leqq \theta_1, \tag{1.3}$$

where the coefficients $P_i(t)$ $(i=0, 1,\dots)$ are holomorphic in $|t| \leqq t_1$, such that the