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On Oriented G-Manifolds of Baas-Sullivan Type

Dedicated to Professor A. Komatu on his 70th birthday

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Introduction

N. A. Baas [1] has studied a bordism theory based on manifolds with a certain type of singularities, by reformulating a theory due to D. Sullivan.

The purpose of this paper is to study corresponding equivariant oriented bordism theories. By the same way as [1; § 2], we can define the notion of oriented $\mathscr{S}_n$-manifolds for each singularity class $\mathscr{S}_n=\{P_0=\mathrm{pt}, P_1,\dots, P_n\}$ of closed oriented manifolds, and those with G-actions for each finite group G while G acts trivially on P_i. Thus we obtain naturally a bordism group $\Omega(\mathscr{S}_n)_*^G(X, Y)$ based on oriented $\mathscr{S}_n$-manifolds with free G-actions for each pair (X, Y) of G-spaces. When $n=0$, $\Omega(\mathscr{S}_0)_*^G(-)$ coincides with the usual equivariant bordism group $\Omega_*^G(-)$, due to Conner-Floyd [3], based on (closed) oriented manifolds with free G-actions.

We study in § 1 (and § 5) some basic properties of oriented $\mathscr{S}_n$-manifolds and the above bordism group, and obtain an exact sequence in Theorem 1.16 which is similar to that in [1; Th. 3.2]. In case that $G=\boldsymbol{Z}_p$ for odd prime p, we can define in § 2 the Smith homomorphism, and extend some results on $\Omega_*^G(-)$, due to P. E. Conner [2] and C. M. Wu [8], to those on $\Omega(\mathscr{S}_n)_*^G(-)$ for each n. Furthermore, we obtain in § 3 a theorem on the Ω_*-module structure of $\Omega(\mathscr{S}_1)_*^{\boldsymbol{Z}_p}$ for an odd dimensional manifold P_1 of Dold type. Finally in § 4, we study oriented $\mathscr{S}_n$-manifolds with semi-free G-actions.

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§ 1. Oriented singular $(G, \mathscr{S}_n)$-manifolds

Throughout this paper, we will work in the category of compact *oriented* smooth manifolds, and we allow the manifolds to have general corners, (see [4] for manifolds with corners, whose coordinate neighborhoods are defined by using open subsets of $\{(x_1,\dots, x_m)\in \boldsymbol{R}^m \mid x_1\geqq 0,\dots, x_m\geqq 0\}$).

DEFINITION 1.1. Let an oriented manifold V and oriented submanifolds