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Exterior Functions and Strictly Continuous Homomorphisms in the Algebra of Bounded Analytic Functions

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Let G be a bounded region in the complex plane, and let $\beta(G)$ be the algebra of bounded analytic functions on G, in the strict topology. The strict topology was introduced by Buck (see [B]) — for a survey of this and related matters see [R]. Suffice it to say that the strict topology is the strongest topology on the bounded holomorphic functions for which a sequence is convergent if and only if it is uniformly bounded and pointwise convergent to its limit (see [RR], Corollary, p. 172). A function $f \in \beta(G)$ is called exterior (see [RS], p. 72) when the principal ideal generated by f is dense in $\beta(G)$. Examples can be found (we give one later) of bounded regions G with $0 \in \partial G$ such that $f(z)=z$ is not exterior. On the other hand, if ∂G consisted of isolated Jordan curves, say, then this function z would certainly be exterior (see [RS], Theorem 5.17). In this paper, we consider $f(z)=z-\lambda$, $\lambda \in C$, and discuss the principal ideal that it generates. We prove that aside from the trivial case where $\lambda \notin \bar{G}$, so that $(z-\lambda)$ is a unit in $\beta(G)$, there are exactly three possibilities. We conclude by proving that $(z-\lambda)$ is exterior if and only if there is no strictly continuous multiplicative linear functional in the fiber over λ. We rely heavily in our exposition on results in Gamelin and Garnett's paper [GG] which appeared at about the time our work in this area was being done.

DEFINITION. A point $\lambda \in \partial G$ is called an essential boundary point of G if there is an $f \in \beta(G)$ such that for no region W that contains λ does there exist an extension of f in $\beta(G \cup W)$. (See [RUD], p. 333.)

THEOREM. *Let G be a bounded region, all of whose boundary points are essential. Let $\beta(G)$ be the algebra of all bounded analytic functions on G, in the strict topology. For $\lambda \in \bar{G}$, let*

$$I(\lambda) = \{(z - \lambda)f : f \in \beta(G)\}.$$

Then exactly one of the following possibilities holds:

1) *$I(\lambda)$ is dense in $\beta(G)$; that is $(z-\lambda)$ is exterior.*
2) *$I(\lambda)$ is closed in $\beta(G)$ and has codimension 1 in $\beta(G)$.*

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