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Unitary representations and kernel functions associated with boundaries of a bounded symmetric domain

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1. Introduction

Let $\mathscr{D}$ be the open unit disc in $\boldsymbol{C}$ and let $\mathscr{B}$ be its boundary. Then the group $G=SU(1, 1)$ of all two-by-two complex matrices of the form $\begin{pmatrix} a & b \\ \bar{b} & \bar{a} \end{pmatrix}$ with $|a|^2-|b|^2=1$ acts transitively both on $\mathscr{D}$ and $\mathscr{B}$ by linear fractional transformations

$$z \longrightarrow g\cdot z = \frac{az+b}{\bar{b}z+\bar{a}} \qquad \text{if} \quad g = \begin{pmatrix} a & b \\ \bar{b} & \bar{a} \end{pmatrix}.$$

The discrete series representations of G can be realized on Hilbert spaces of holomorphic (or anti-holomorphic) functions on $\mathscr{D}$, while the principal continuous series representations can be realized on $L^2(\mathscr{B})$. Every member of the principal continuous series representations of G is irreducible except one, say V, which is given by

$$(V(g)f)(u) = j(g^{-1}, u)^{1/2} f(g^{-1}\cdot u), \quad f\in L^2(\mathscr{B}),\ g\in G,\ u\in\mathscr{B}$$

where $j(g^{-1}, u)$ denotes the complex Jacobian of the holomorphic map $z\to g^{-1}\cdot z$ at u ($j(g^{-1}, u)=(\bar{b}u+\bar{a})^{-2}$ if $g^{-1}=\begin{pmatrix} a & b \\ \bar{b} & \bar{a} \end{pmatrix}$).

The so-called holomorphic discrete series representations of G are parametrized by the integers $n\geq 2$, and the n-th representation T_n is realized on the Hilbert space

$$H_n = \left\{\text{holomorphic functions } f \text{ on } \mathscr{D};\ \int_{\mathscr{D}} |f(z)|^2(1-|z|^2)^{n-2}dxdy < \infty\right\}$$

with group action

$$(T_n(g)f)(z) = j(g^{-1}, z)^{n/2} f(g^{-1}\cdot z), \quad f\in H_n,\ g\in G,\ z\in\mathscr{D}.$$

Note that in the case $n=1$ we have $H_1=\{0\}$. However, one can associate to the integer $n=1$ a representation of G that is similar in appearance to those above. Indeed, if we let $H^2(\mathscr{D})$ be the Hardy space for $\mathscr{D}$, i.e.,