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Necessary and sufficient condition for eventual decay of oscillations in general functional equations with delays

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1. Introduction

Our main purpose in this paper is to study the equation

$$(r(t)y'(t))^{(n-1)} + a(t)h(y(g(t))) = f(t) \tag{1}$$

and present a necessary and sufficient condition so that all oscillatory solutions of equation (1) converge to zero asymptotically.

In [2, 3], this author showed that subject to

$$\int^{\infty} t^{n-2}\,|a(t)|dt < \infty \tag{2}$$

$$\int^{\infty} t^{n-2}\,|f(t)|dt < \infty \tag{3}$$

and boundedness of $(t^{n-k})/r(t)$, $0 \leq k < 1$ for $t \in [T, \infty)$, $T > 0$ all oscillatory solutions approach zero as $t \to \infty$. There are examples given in [2] to show that condition on $r(t)$ cannot be weakened. This restriction on $r(t)$ eliminates a very important class of equations of type (1) that requires $\int^{\infty} 1/r(t)dt = \infty$. We find a set of conditions in Theorem 3.2 which essentially ensure that all oscillatory solutions of (1) eventually vanish while retaining $\int^{\infty} 1/r(t)dt = \infty$. We, then, use this theorem to find a necessary and sufficient condition to accomplish the stated goal of this work in section 4.

2. Definition and assumptions

Unless otherwise stated, following assumptions apply throughout this work:

(i) $g(t)$, $r(t)$, $a(t)$, $f(t)$ and $h(t)$ are $R \to R$ and continuous, R being the real line;

(ii) $r(t) > 0$, $r'(t) \geq 0$ for $t \geq t_0$ where $t_0 > 0$ will be assumed fixed;

(iii) $th(t) > 0$, $t \neq 0$ and there exists an $m > 0$ such that $\dfrac{h(t)}{t} \leq m$ for $t \neq 0$;