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On errors in the numerical solution of ordinary differential equations by step-by-step methods

Hisayoshi SHINTANI
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1. Introduction

Consider the initial value problem

$$y' = f(x, y) \qquad (a \leqq x \leqq b), \tag{1.1}$$

$$y(a) = y_0, \tag{1.2}$$

where $f(x, y)$ is sufficiently smooth in $I \times R$, $I=[a, b]$ and $R=(-\infty, \infty)$. Denote by $y(x)$ the solution of this problem and for a positive constant h_0 let

$$x_j = a + jh \ (j = 0, 1,\dots, N), \quad h = (b-a)/N \leqq h_0. \tag{1.3}$$

We consider the case where the approximate values y_m of $y(x_m)$ $(m=k, k+1,\dots, N)$ are obtained by the k-step method [2]

$$\textstyle\sum_{j=0}^{k} \alpha_j y_{n+j} = h\Phi(x_n, y_n,\dots, y_{n+k}; h) \qquad (n = 0, 1,\dots, N-k), \tag{1.4}$$

where α_j $(j=0, 1,\dots, k)$ are real constants and $\alpha_k=1$. The method (1.4) includes one-step methods, linear multistep methods, hybrid methods, pseudo-Runge-Kutta methods and so on.

In Section 3 for sufficiently smooth $\Phi(x, u_0,\dots, u_k; v)$ we study the asymptotic behavior of errors

$$e_j = y_j - y(x_j) \qquad (j = 0, 1,\dots, N) \tag{1.5}$$

as $h \to 0$. In Section 4 the local truncation error is approximated and Milne's device in the predictor-corrector method is justified under certain conditions. In Section 5 we are concerned with the approximate computation of errors and illustrate the method by numerical examples.

2. Preliminaries

2.1. Assumptions

For simplicity the dependence of Φ on f is not expressed explicitly. Let