

Relative invariants of prehomogeneous vector spaces and a realization of certain unitary representations I

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1. Introduction

The purpose of this paper is to describe the unitary representations of some real semi-simple Lie groups on the spaces of solutions for certain differential equations.

We are concerned with a Lie group G satisfying the following two conditions:

1. If \mathfrak{g} is the Lie algebra of G , then \mathfrak{g} has a \mathbf{Z} -graded decomposition $\mathfrak{g} = \mathfrak{g}_{-1} + \mathfrak{g}_0 + \mathfrak{g}_1$.
2. If G_0 is the subgroup of G corresponding to \mathfrak{g}_0 , then the real prehomogeneous vector space (G_0, \mathfrak{g}_1) possesses a relative invariant.

We take the suitable regular or singular orbits of (G_0, \mathfrak{g}_1) and construct the Hilbert spaces of holomorphic functions on G/K by means of the Fourier-Laplace transform of the functions supported on these orbits. To construct the irreducible and unitary representations of G we use R. A. Kunze's reproducing kernel method [11]. The key of this construction is the Fourier transform of the relative invariant of (G_0, \mathfrak{g}_1) , which was also the key in [1], [14], and is studied from a new point of view in [9].

We make some bibliographic comments.

In [4], [5], and [6] Harish-Chandra constructed a certain class of representations of a simply connected real semi-simple Lie group G whose associated symmetric space G/K is hermitian. This class includes the holomorphic discrete series. Rossi and Vergne [12] and Wallach [15], [16] have studied the analytic continuation of the holomorphic discrete series for the scalar case. Furthermore in [12] it is shown that certain of these representations can be realized on the Hardy type Hilbert spaces associated with various boundary orbits in G/K . For the general case similar results were obtained by Inoue [7]. For the groups associated with classical hermitian symmetric spaces of tube type, all these representations were obtained by Gross and Kunze [2], [3] by considering the generalized gamma functions. For the conformal group $SU(2, 2)$ Jakobsen and Vergne constructed the irreducible unitary representations on the solution spaces