HIROSHIMA MATH. J.
11 (1981), 369–378

Programmings with constraints of convex processes

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(Received January 13, 1981)

§ 1. Introduction

Let X and Y be real linear spaces which are in duality with respect to a bilinear functional $((\ ,\ ))_1$ and let Z and W be real linear spaces which are in duality with respect to a bilinear functional $((\ ,\ ))_2$. A standard linear programming problem is:

$$(\mathrm{P}_1)\quad \text{Find } M_1 = \inf\{((x, y_0))_1;\ x \in P,\ Ax - z_0 \in Q\},$$

where $y_0 \in Y$, $z_0 \in Z$, P and Q are weakly closed convex cones in X and Z respectively and A is a weakly continuous linear mapping from X to Z.

A dual problem of (P_1) is:

$$(\mathrm{D}_1)\quad \text{Find } M_1^* = \sup\{((z_0, w))_2;\ w \in Q^\circ,\ y_0 - A^*w \in P^\circ\},$$

where P° and Q° are the polar sets of P and Q respectively and A^* is the adjoint mapping of A.

Kretschmer showed

THEOREM 0 ([4; Theorem 3]). (a) *If the set* $H = \{(Ax - z, r + ((x, y_0))_1);\ x \in P,\ z \in Q,\ r \geq 0\}$ *is weakly closed in* $Z \times R$ *and* M_1 *or* M_1^* *is finite, then* $M_1 = M_1^*$ *and there exists an* $x_0 \in P$ *such that* $Ax_0 - z_0 \in Q$ *and* $((x_0, y_0))_1 = M_1$.

(b) *If there exists an element* $w_0 \in Q^\circ$ *such that* $y_0 - A^*w_0$ *is contained in the interior of* P° *with respect to the Mackey topology, then* H *is weakly closed.*

Later on, Fan [2 and 3] dealt with the case where one of P and Q is merely closed convex. Under some conditions, he showed a duality between (P_1) and (D_2):

$$(\mathrm{D}_2)\quad \sup\{(((z_0, w))_2 - 1)/r;\ r > 0,\ w \in Q^\circ,\ ry_0 - A^*w \in P^\circ\}.$$

Furthermore, Levin-Pomerol [5] and Zălinescu [9] were interested in the problems which contain a positively homogeneous functional:

$$(\mathrm{P}_3)\quad \inf\{((x, y_0))_1;\ x \in P,\ (Ax - C) \cap Q \neq \emptyset\},$$

where C is a weakly compact convex subset of Z,

$$(\mathrm{D}_3)\quad \sup\{(g_C(w) - 1)/r;\ r > 0,\ w \in Q^\circ,\ ry_0 - A^*w \in P^\circ\},$$