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On CW cospectra

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Introduction

E. L. Lima [3] defined a direct spectrum $\{E_i, \phi_i: E_i \to E_{i+1}\}$ and an inverse spectrum $\{F_i, \psi_i: F_{i+1} \to F_i\}$. The former has been developed by many authors into the theory of CW spectra, which is now the basic notion in the cohomology theory ([1], [6], [7], [8]). In this paper, we shall define the notion of CW cospectra corresponding to the latter, and argue the homotopy category of CW cospectra by treating it as dual to that of CW spectra.

In this paper, a CW complex is called a nice complex if each cell is a subcomplex, and a map between nice complexes is called a nice map if each cell is mapped onto a subcomplex. By using the category NCW of nice complexes and nice maps, we define a *CW cospectrum E* as a collection

$$E = \{E_n, \varepsilon_n: E_{n+1} \longrightarrow SE_n \mid n \in Z\}$$

in NCW where S denotes the suspension and ε_n is the projection shrinking a subcomplex of E_{n+1} to $*$, and a *map*

$$f: E = \{E_n, \varepsilon_n\} \longrightarrow F = \{F_n, \varepsilon'_n\}$$

between CW cospectra is a collection of $f_n: E_n \to F_n/F'_n$ in NCW commuting with ε_n and ε'_n, where $F' = \{F'_n\}$ is a null subcollection of F, (see Definitions 1.1, 1.4 and 1.10). Further, a homotopy is a map $h: E \wedge I^+ \to F$ where $I^+ = \{*\} \cup [0, 1]$ (disjoint union) and $(E_n \wedge I^+)_n = E_n \wedge I^+$ (see Definition 1.14).

Thus, we obtain the *homotopy category of CW cospectra*. Furthermore, by considering the notion of cells in a CW cospectrum, we define a CW cospectrum E of finite type and the *cohomotopy group*

$$\pi^n(E) = [E, \Sigma^n S^0] \quad \text{(homotopy set)} \qquad \text{for any} \quad n \in Z$$

where $(\Sigma^n S^0)_i = *$ $(i < -n)$, $= S^{n+i}$ $(i \geq -n)$, (see Definitions 2.1, 2.4 and 3.3). Then, we have the following

THEOREM 3.5. *Assume that a CW cospectrum E of finite type satisfies $\pi^n(E) = 0$ for any n. Then, E is contractible in the homotopy category of CW cospectra.*

COROLLARY 3.8. *Let E be a CW cospectrum of finite type. Then, there is*