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Essential self-adjointness of Schrödinger operators with potentials singular along affine subspaces

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1. Introduction

The aim of this paper is to study the essential self-adjointness of a Schrödinger operator $-\Delta+q(x)$ acting in $L^2(\mathbf{R}^m)$, $m\geqq 1$, with the domain $C_0^\infty(\mathbf{R}^m\backslash F)$, where F is the union of at most countable number of k_α-dimensional $(0\leqq k_\alpha\leqq m-1)$ affine subspaces S_α $(\alpha\in A)$ in $\mathbf{R}^m$ which satisfy

$$r = \inf\{\operatorname{dist}(S_\alpha, S_\beta);\ \alpha,\ \beta\in A,\ \alpha\neq\beta\} > 0.$$

Here $\operatorname{dist}(S_\alpha, S_\beta)$ denotes the distance from S_α to S_β.

This study is motivated by a theorem proved by B. Simon [6], which is a generalization of the results of H. Kalf and J. Walter [1] and U. W. Schmincke [5]. In this theorem of Simon, which corresponds to the case of $F=\{0\}$, it is assumed that the potential $q=q_1+q_2$ is a real-valued function with $q_1\in L^2_{\mathrm{loc}}(\mathbf{R}^m\backslash\{0\})$ and $q_2\in L^\infty(\mathbf{R}^m)$ such that

$$q_1(x) \geqq -(1/4)m(m-4)|x|^{-2} \qquad (x\in\mathbf{R}^m\backslash\{0\}).$$

We extend this result to the case of the general F as stated above. The following is our theorem.

THEOREM. *Set $\Omega=\mathbf{R}^m\backslash F$ and let $a_j\in C^1(\Omega)$ $(1\leqq j\leqq m)$, $q_1\in L^2_{\mathrm{loc}}(\Omega)$ and $q_2\in L^\infty(\mathbf{R}^m)$ be real-valued functions. Assume that for some ε $(0<\varepsilon<r/2)$, q_1 satisfies the following conditions*:

(C.1) *For each $\alpha\in A$*

$$q_1(x) \geqq -(1/4)(m-k_\alpha)(m-k_\alpha-4)[\operatorname{dist}(x, S_\alpha)]^{-2}$$

whenever $0<\operatorname{dist}(x, S_\alpha)<\varepsilon$.

(C.2) *q_1 is bounded from below on*

$$\textstyle\bigcap_{\alpha\in A}\{x\in\mathbf{R}^m;\ \varepsilon\leqq\operatorname{dist}(x, S_\alpha)\}.$$

Let $q=q_1+q_2$. Then the symmetric operator T acting in $L^2(\mathbf{R}^m)$ defined by

$$T = -\textstyle\sum_{j=1}^m(\partial/\partial x_j - ia_j(x))^2 + q(x), \quad D(T) = C_0^\infty(\Omega),$$