HIROSHIMA MATH. J.
11 (1981), 255–273

Construction of one-dimensional classical dynamical system of infinitely many particles with nearest neighbor interaction

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(Received September 18, 1980)

§1. Introduction

In the investigation of the time evolution of a system of infinitely many particles which can be described by Newton's equations of motion, the first problem is to construct a dynamical system, more precisely, to determine a class of initial configurations for which equations of motion have solutions; the next problem is to investigate statistical mechanical properties of the dynamical system such as ergodicity. As for the construction of dynamical systems many results were obtained ([1], [2], [4]–[7]); especially in [5] and [6] ν-dimensional systems with long range interactions were treated. However, an explicit description of a class of initial configurations for which equations of motion have solutions was given only in the works of Dobrushin and Fritz ([1], [2]) in 1977.

We consider a system of infinitely many classical particles moving on the real line $\mathbf{R}$ in such a way that each particle is under interaction (repulsive force) only with its two right and left neighboring particles (the precise description of our model is given in §2). In this paper we construct the dynamical system for our model starting with a class $\mathscr{X}_\gamma$ of initial configurations, $0 \leqq \gamma < 1$. The class $\mathscr{X}_\gamma$ can be described as in [1]; in fact, it is given by (2.8) in §2. The uniqueness problem is also considered. The Gibbs states for our model become renewal measures ([3]), and from this fact it will follow that the class $\mathscr{X}_\gamma$ has full measure with respect to the Gibbs states. In this sense $\mathscr{X}_\gamma$ may be considered sufficiently wide.

The author would like to express his gratitude to Professors H. Tanaka and H. Murata for their constant encouragement, and to Professor T. Shiga who kindly pointed out to the author the results on renewal measures in [3].

§2. Definitions and results

In this section we give the definitions and notations used throughout this paper and state the theorems.

Given a potential function $\Phi(r)$, $r > 0$, we consider the one-dimensional system of infinitely many (indistinguishable) particles moving according to the