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Normal limits, half-spherical means and boundary measures of half-space Poisson integrals

David H. ARMITAGE

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1. Introduction and results

Let D denote the Euclidean half-space $\boldsymbol{R}^n \times (0, +\infty)$, where $n \geq 1$, and let ∂D denote the Euclidean boundary of D. Arbitrary points of D and ∂D are denoted by $M=(X, x)$ and $P=(T, 0)$, respectively, where $X, T \in \boldsymbol{R}^n$ and $x \in (0, +\infty)$. We write $|M|$ for the Euclidean norm of M.

If μ is a signed measure on ∂D such that

$$\int_{\partial D} (1+|P|)^{-n-1} d|\mu|(P) < +\infty, \tag{1}$$

then the Poisson integral I_μ of μ is defined in D by the equation

$$I_\mu(M) = 2(s_{n+1})^{-1} \int_{\partial D} x|M-P|^{-n-1} d\mu(P),$$

where s_{n+1} is the surface-area of the unit sphere in $\boldsymbol{R}^{n+1}$. The condition (1) is necessary and sufficient for I_μ to be harmonic in D (see Flett [5], Theorem 6), and we say that a measure μ on ∂D is of class $\mathscr{F}$ if (1) is satisfied. If, further, μ is non-negative, we write $\mu \in \mathscr{F}^+$.

For each point P of ∂D and each positive number r, we write

$$\sigma(P, r) = \{M \in D\colon |M-P| = r\},$$
$$\tau(P, r) = \{Q \in \partial D\colon |Q-P| < r\},$$

and we denote surface-area measure on $\sigma(P, r)$ or ∂D by s.

If h is the difference of two non-negative harmonic functions in D, then the function $\mathscr{M}(h, P, \cdot)$, defined on $(0, +\infty)$ by

$$\mathscr{M}(h, P, r) = r^{-n-2} \int_{\sigma(P,r)} x h(M) ds(M),$$

is real-valued and continuous on $(0, +\infty)$ and is bounded on any interval of the form $[a, +\infty)$, where $a>0$. In the case where $h \geq 0$ in D, the mean $\mathscr{M}(h, P, r)$ is a decreasing function of r and a convex function of r^{-n-1} on $(0, +\infty)$. Papers dealing with this mean include those of Dinghas [2], [3], Kuran [9], [10] and the author [1].