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Stationary pattern of some density-dependent diffusion system with competitive dynamics

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1. Introduction

Recently density-dependent diffusion equations have been extensively investigated. The first interesting equation is a model of gas flow through a homogeneous porous medium, which is of the form

$$(1) \qquad u_t = \Delta(u^m) \qquad (m > 1),$$

where u describes the denisty of the gas. Because of the degeneracy of the diffusivity at $u=0$, there is an attractive phenomenon such as the finite speed of propagation of disturbances (see, for example, Aronson [2] and its bibliography). The second model is one species population model similar to (1),

$$(2) \qquad u_t = \Delta(\phi(u)) + f(u),$$

where u means the population density, $\phi(u)$ is a monotone increasing function for $u>0$ with $\phi(0)=0$. This nonlinearity implies that dispersal is influenced by local population pressure and $f(u)$ is the population supply such as Fischer's type (Gurtin and MacCamy [4] and Newman [10]).

For systems of equations as an extension of (2), Shigesada et al. [14] proposed a model of two competing species with self- and cross-population pressures so as to discuss the problem of spatial segregation

$$(3) \qquad \begin{cases} u_t = \Delta\{(d_{11}+d_{12}v)u\} + (R_1-a_1u-b_1v)u, \\ v_t = \Delta\{(d_{22}+d_{21}u)v\} + (R_2-a_2v-b_2u)v, \end{cases}$$

where d_{ij}, R_i, a_i and $b_i (i, j=1, 2)$ are positive constants or zero. When $d_{ij}>0 (i\neq j)$, the population pressure of each species is exerted on the other and raises its dipersive force. Aronson [2] has also proposed a similar population model of prey and predator interaction, which is represented by

$$(4) \qquad \begin{cases} u_t = \rho u(1-u/K) - \alpha uv, \\ v_t = \Delta\{\psi(u)v\} - \mu v + \gamma uv, \end{cases}$$

where ρ, K, α, μ, γ are positive constants and