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On strong oscillation of even order differential equations with advanced arguments

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This paper is concerned with the oscillatory behavior of solutions of linear functional differential equations of the form

$$x^{(n)}(t) + p(t)x(g(t)) = 0, \tag{1}$$

where n is even, $p: [a, \infty)\to(0, \infty)$ is continuous, $g: [a, \infty)\to R$ is continuously differentiable, $g'(t)>0$ and $\lim_{t\to\infty} g(t)=\infty$. By a proper solution of equation (1) is meant a function $x: [T_x, \infty)\to R$ which satisfies (1) for all sufficiently large t and $\sup\{|x(t)|; t\geq T\}>0$ for any $T\geq T_x$. A proper solution of (1) is called oscillatory if it has arbitrarily large zeros, and nonoscillatory otherwise. Equation (1) is said to be oscillatory if all of its solutions are oscillatory; otherwise equation (1) is said to be nonoscillatory. Equation (1) is said to be *strongly oscillatory* or *strongly nonoscillatory* according as the equation

$$x^{(n)}(t) + kp(t)x(g(t)) = 0 \tag{2}$$

is oscillatory or nonoscillatory for every $k>0$.

Recently Naito [2] has proved the following theorem for the strong oscillation and nonoscillation of retarded equations of the form (1).

THEOREM 1. *Suppose that* $g(t)\leq t$ *for* $t\geq a$ *and*

$$\liminf_{t\to\infty} g(t)/t > 0. \tag{3}$$

Equation (1) *is strongly oscillatory if and only if*

$$\limsup_{t\to\infty} t\int_t^\infty s^{n-2}p(s)ds = \infty, \tag{4}$$

and equation (1) *is strongly nonoscillatory if and only if*

$$\lim_{t\to\infty} t\int_t^\infty s^{n-2}p(s)ds = 0. \tag{5}$$

A question naturally arises as to what will happen for the advanced case of (1). The purpose of this paper is to give an answer to this question by showing that a similar conclusion holds in this case.