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Positive solutions of semilinear second order elliptic equations in exterior domains

Nichiro KAWANO and Manabu NAITO

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1. Introduction

This paper is concerned with positive solutions of the semilinear elliptic equation

$$\Delta u - \phi(x)|u|^{\gamma}\operatorname{sgn} u = 0 \tag{1}$$

in exterior domains, where ϕ is a positive continuous function, γ is a positive constant and $\Delta = \sum_{i=1}^{n} \partial^2/\partial x_i^2$ is the Laplace operator in Euclidean space R^n. Equation (1) is called superlinear or sublinear according as $\gamma > 1$ or $0 < \gamma < 1$.

Recently equations including

$$\Delta u + \phi(x)|u|^{\gamma}\operatorname{sgn} u = 0 \tag{2}$$

have been considered by Noussair and Swanson [6, 8] and effective conditions for (2) to have positive solutions in exterior domains have been established. For other related results with regard to (2) the reader is referred to Kitamura and Kusano [3], Noussair [4] and Noussair and Swanson [5, 7].

Our purpose here is to discuss the existence and asymptotic behavior of positive solutions, defined in exterior domains, of equation (1) which has little been studied in the literature. Employing the techniques of Noussair and Swanson, we reduce the multi-dimensional problem under study to the problem of one dimension and make extensive use of known results on the existence and asymptotic behavior of positive solutions of ordinary differential equations of the form

$$y'' - p(t)|y|^{\gamma}\operatorname{sgn} y = 0, \qquad p(t) > 0. \tag{3}$$

In Section 2 we prove a basic existence theorem for positive solutions of equation (1). We distinguish the superlinear case (Section 3) and the sublinear case (Section 4), and establish in each case effective sufficient conditions under which equation (1) possesses positive solutions having various asymptotic properties as $|x| \to \infty$.