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On nonstationary solutions of the Navier-Stokes equations in an exterior domain

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Introduction

Let us consider a moving body in a viscous incompressible fluid filling the whole space R^3. If we describe the fluid motion by using a coordinate system attached to the body, we obtain the exterior problem for the Navier-Stokes equations:

$$(*)\qquad \begin{aligned} \partial u/\partial t - \Delta u + (u, \nabla)u &= f - \nabla p && \text{in } D \times (0, T), \\ \operatorname{div} u &= 0 && \text{in } D \times (0, T), \\ u(x, t) &= u^*(x, t) && \text{on } S \times (0, T), \\ u(x, t) &\to u_\infty(t) && \text{as } |x| \to \infty, \\ u(x, 0) &= a(x) && \text{in } D. \end{aligned}$$

Here D is the exterior to the body with the boundary S which we assume to be smooth; $u=\{u^j(x, t)\}_{j=1}^3$ and $p=p(x, t)$ denote, respectively, the unknown velocity and pressure, while $f=\{f^j(x, t)\}_{j=1}^3$ and $a=\{a^j(x)\}_{j=1}^3$ denote, respectively, the given external force and initial velocity. u^* and u_∞ are given boundary data. For this problem, Hopf [16] proved the existence of a square-summable weak solution, when $u^*=u_\infty=0$, for an arbitrary square-summable initial velocity.

On the other hand, in the case of stationary flow, i.e., when $\partial u/\partial t=0$, $u^*=0$ and $u_\infty=\text{const.}$, Finn [4], [5], [6] proved the existence of a solution, called a physically reasonable solution, which exhibits a phenomenon of wake. Moreover, in [3] he showed that if $u(x)$ is such a solution and if the force exerted to the body by the flow does not vanish, then $u(x)-u_\infty$ is not square-summable over D.

In view of the above result, it seems reasonable to seek a solution of the problem $(*)$ in a class of functions $u(x, t)$ such that $u(x, t)-u_\infty(t)$ is not square-summable over D. This problem was discussed by Heywood in a series of papers [12], [13], [14], [15]. He showed a local existence result in the class of functions with finite Dirichlet integral by using a variant of the Faedo-Galerkin approximation developed by Hopf [16], Kiselev and Ladyzhenskaya [18] and Prodi [28]. However, he assumes that the initial function a be square-summable in proving the existence of a global solution; see [15, Th. 6].