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Ascendancy in locally finite groups

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Introduction

In the paper [2], we introduced the notion of weakly ascendant subgroups which is weaker than that of ascendant subgroups, and mainly investigated the relationship of these notions in generalized solvable groups. Recently in the papers [1, 3], ascendancy has been studied in locally solvable, ideally finite Lie algebras.

In this paper, following the line of the papers [1, 3] we shall investigate ascendancy and weak ascendancy in locally finite groups, especially in groups which correspond to locally solvable, ideally finite Lie algebras.

Let G be a group and H be a subgroup of G. In Section 2 we shall show that when $G \in \mathrm{L}(H)\mathfrak{F}$, H is weakly ascendant in G if and only if H is ω-step weakly ascendant in G (Theorem 1). In Section 3 we shall show as the main result of the paper that when $G \in \mathrm{L}(\lhd)(\mathrm{E}\mathfrak{A} \cap \mathfrak{F})$, H is weakly ascendant in G if and only if H is ascendant in G and if and only if H is ω-step ascendant in G (Theorem 3). In Section 4 we shall study the cases where G belongs to $\mathrm{L}(\mathrm{sn})\mathrm{E}\mathfrak{A}$ and $\mathrm{L}(\mathrm{asc})\acute{\mathrm{E}}\mathfrak{A}$. In Section 5 we shall present some characterizations of the class of groups $\mathrm{L}(\lhd)(\mathrm{E}\mathfrak{A} \cap \mathfrak{F})$ (Theorem 5).

1.

Let G be a group. If X, Y are non-empty subsets of G, we denote by $[X, Y]$ the set of all $[x, y] = x^{-1}y^{-1}xy$ with $x \in X$ and $y \in Y$ and we write $[X, {}_0Y] = X$, $[X, {}_{n+1}Y] = [[X, {}_nY], Y]$ for an integer $n \geq 0$.

If H is respectively an ascendant subgroup, a σ-step ascendant subgroup and a subnormal subgroup of G, we as usual write

$$H \operatorname{asc} G, \quad H \lhd^{\sigma} G \quad \text{and} \quad H \operatorname{sn} G,$$

where σ is an ordinal.

Let $H \leq G$. As in [2], we call H a σ-step weakly ascendant subgroup of G, if there is an ascending series $(S_\alpha)_{\alpha \leq \sigma}$ of subsets of G satisfying the following conditions:

(a) $S_0 = H$ and $S_\sigma = G$.

(b) If α is any ordinal $< \sigma$, then $u^{-1}Hu \subseteq S_\alpha$ for any $u \in S_{\alpha+1}$.