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A theorem on splitting of algebraic vector bundles and its applications

Dedicated to Professor Yoshikazu Nakai on his 60th birthday

Hideyasu SUMIHIRO
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0. Introduction

Let E be an algebraic vector bundle on a smooth projective algebraic scheme X defined over an algebraically closed field (arbitrary characteristic). Then it is known that after a suitable succession of blowing ups of $X, f: X' \to X, f^*(E)$ has a splitting of line bundles on X', i.e., there is a filtration of subbundles of $f^*(E)$ $F_0 \supset \cdots \supset F_r = 0$ ($r = \mathrm{rank}\, E$) such that every quotient F_i/F_{i+1} $(0 \leqq i \leqq r-1)$ is a line bundle on X' (cf. [4]). In this paper, we shall prove another simple theorem on splitting of line bundles of algebraic vector bundles (cf. Theorem 2.1): Let E be an algebraic vector bundle on a smooth quasi-projective algebraic scheme defined over an algebraically closed field (arbitrary characteristic). Then there exists a finite and faithfully flat morphism $f: X' \to X$ such that $f^*(E)$ has a splitting of line bundles on X'. Hence we can prove the following (cf. Theorem 3.2) as a corollary: Let Z be an algebraic cycle of codim $= p$ on a smooth projective algebraic scheme X. Then there is a finite faithfully flat morphism $f: X' \to X$ such that $(p-1)!f^*(Z) = \sum \pm D_1 \cdots D_p$ (rat. equiv.), where D_k are divisors on X'. Hence in particular, $(p-1)!f^*(Z)$ is smoothable. Theorem 3.2 seems to be a useful fact to study algebraic cycles because it says that if a problem on algebraic cycles is not changed after multiplication of integers and pull back of finite faithfully flat morphisms, then we have only to consider the cycles Z of the forms $\sum \pm D_1 \cdots D_p$, where D_k are divisors on X. After introducing the notion of very ample vector bundles and studying their properties, we shall prove the above theorems.

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1. Very ample vector bundles

In [2], R. Hartshorne has introduced the notion of ampleness of algebraic vector bundles. Since then, we have obtained several useful algebro-geometric results using ample vector bundles. In this section, we shall define very ample vector bundles on algebraic schemes and study their properties.