Hiroshima Math. J.
12 (1982), 249–258

A note on excellent forms

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(Received September 18, 1981)

The notion of excellent quadratic forms was first introduced in [3] by M. Knebusch and some basic properties were investigated there. However it seems to the authors that the most important theorem, so to speak '*the structure theorem of excellent forms*', has not been known yet. The main purpose of this paper is to give theorems of this sort (cf. Theorem 2.1, Theorem 2.4).

§ 1. Definitions and notations

Throughout this paper, a field always means a field of characteristic different from 2. Let $\dot{k}$ denote the multiplicative group of a field k. The Witt decomposition theorem says that any quadratic form φ is decomposable into $\varphi_h \perp \varphi_a$, where φ_a is anisotropic, and $\varphi_h \cong mH$ is hyperbolic. Here, φ_a is uniquely determined up to isometry by φ, and so we speak of φ_a as the 'anisotropic part' of φ. The integer m above is also uniquely determined by φ, and will be called the '*Witt index*' of φ.

For a form φ over k and an element $a \in \dot{k}$, we shall abbreviate $\langle a \rangle \otimes \varphi$ to $a\varphi$ if there is no fear of confusion. For any form φ over k, we denote the set $\{a \in \dot{k} \mid \varphi \text{ represents } a\}$ by $D_k(\varphi)$ and the set $\{a \in \dot{k} \mid a\varphi \cong \varphi\}$ by $G_k(\varphi)$. The latter is always a subgroup of $\dot{k}$. When φ and ψ are similar, namely $\varphi \cong a\psi$ for some $a \in \dot{k}$, we write $\varphi \approx \psi$. We say that ψ is a subform of φ, and write $\psi < \varphi$, if there exists a form χ such that $\varphi \cong \psi \perp \chi$. We say that ψ divides φ, and write $\psi \mid \varphi$, if there exists a form χ such that $\varphi \cong \psi \otimes \chi$.

For an n-tuple of elements $(a_1, \dots, a_n)$ of $\dot{k}$, we write $\langle\langle a_1, \dots, a_n \rangle\rangle$ to denote the 2^n-dimensional Pfister form $\otimes_{i=1,\dots,n} \langle 1, a_i \rangle$. Since any Pfister form φ respresents 1, we may write $\varphi \cong \langle 1 \rangle \perp \varphi'$. The form φ' is uniquely determined by φ, and we call φ' the pure subform of φ. A form φ over k is called a Pfister neighbour, if there exist a Pfister form ρ, some a in $\dot{k}$, and a form η with $\dim \eta < \dim \varphi$ such that $\varphi \perp \eta \cong a\rho$. The forms ρ and η are uniquely determined by φ. We call ρ the associated Pfister form of φ, and η the complementary form of φ, and we say more specifically that φ is a neighbour of ρ. A form φ over k is called excellent if there exists a sequence of forms $\varphi = \eta_0, \eta_1, \dots, \eta_t$ $(t \geqq 0)$ over k such that $\dim \eta_t \leqq 1$ and $\eta_i\,(0 \leqq i < t-1)$ is a Pfister neighbour with complementary form η_{i+1}. Each η_r with $0 \leqq r \leqq t$ is uniquely determined by φ, and we call η_r the r-th