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Modified Rosenbrock methods with approximate Jacobian matrices

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1. Introduction

Consider the initial value problem for a stiff system

$$(1.1) \qquad y' = f(y), \quad y(x_0) = y_0,$$

where y is an m-vector and the vector function $f(y)$ is assumed to be sufficiently smooth. Let $y(x)$ be the solution of this problem,

$$(1.2) \qquad x_n = x_0 + nh \qquad (n = 1, 2, \dots, h > 0)$$

and let $J(y)$ be the Jacobian matrix of $f(y)$. We are concerned with the case where the approximations y_j $(j=1, 2, \dots)$ of $y(x_j)$ are obtained by the modified Rosenbrock methods of the form

$$(1.3) \qquad y_{n+1} = y_n + \textstyle\sum_{i=1}^{q} p_i k_i \qquad (n = 0, 1, \dots)$$

which require per step one evaluation of J, k evaluations of f and the solution of a system of m linear equations for q different right hand sides, where

$$(1.4) \quad Mk_i = hf(y_n + \textstyle\sum_{j=1}^{i-1} a_{ij}k_j) + hJ \sum_{j=1}^{i-1} d_{ij}k_j \qquad (i = 1, 2, \dots, q),$$

the matrix $M=I-ahJ$ is nonsingular, $J=J(y_n)$ and a_{ij}, d_{ij} $(j=1, 2, \dots, i-1;$ $i=1, 2, \dots, q)$ and a $(a>0)$ are constants.

Nørsett and Wolfbrandt [3] obtained an A-stable method of order $k+1$ for $k=q=2, 3$. For inexact Jacobian matrices, however, these methods are reduced to methods of lower orders. Steihaug and Wolfbrandt [4] tried to avoid the use of exact Jacobian matrix and considered methods of the form (1.3), called the W-methods, where

$$(1.5) \qquad Wk_i = hf(y_n + \textstyle\sum_{j=1}^{i-1} a_{ij}k_j) + hA \sum_{j=1}^{i-1} d_{ij}k_j \qquad (i = 1, 2, \dots, q),$$

$W=I-ahA$ is nonsingular and A is a matrix approximating J. They have shown that for $q=2^{k-1}$ $(k=2, 3)$ there exists a W-method of order k and that the method of order 2 is $A(0)$-stable under certain conditions.

The first object of this paper is to show that each A-stable modified Rosenbrock method remains A-stable if the Jacobian matrix is approximated with