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The pure braid groups and the Milnor $\bar{\mu}$-invariants of links

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1. The statement of results

In this note, we study a relation between the pure braid groups P_n and the Milnor $\bar{\mu}$-invariants of links, and shall prove the *mod p* residual nilpotence of P_n.

Let

$$X_n = \{(x_1,\dots, x_n) \in \boldsymbol{C}^n \mid x_i \neq x_j \text{ if } i \neq j\}$$

be the configuration space of $\boldsymbol{C}$. Then the symmetric group S_n of degree n acts freely on X_n by the permutation of the coordinates. Let $Y_n = X_n/S_n$ be the quotient space by the action of S_n. Then we have

$$\pi_i(X_n) = \pi_i(Y_n) = 0 \quad (i \geq 2)$$

and the exact sequence

$$1 \longrightarrow \pi_1(X_n) \longrightarrow \pi_1(Y_n) \longrightarrow S_n \longrightarrow 1.$$

DEFINITION 1. $\pi_1(Y_n)$ (resp. $\pi_1(X_n)$) is said to be the *braid group* (resp. the *pure braid group*) of degree n, and is denoted by B_n (resp. P_n).

In fact, B_n coincides with Artin's braid group of the equivalence classes of braids (see [1]).

For any braid $b \in B_n$, let $\hat{b}$ be the closed braid of b (see [1]). If $b \in P_n$, then $\hat{b}$ is a link of n components in S^3.

DEFINITION 2. Put

$$P_{n,q} = \{b \in P_n \mid \bar{\mu}(i_1 \cdots i_k)(\hat{b}) = 0 \quad \text{for any} \quad k \leq q\},$$
$$P_{n,q}^{(p)} = \{b \in P_n \mid \bar{\mu}(i_1 \cdots i_k)(\hat{b}) \equiv 0 \quad mod\, p \quad \text{for any} \quad k \leq q\}$$

where $\bar{\mu}$ is the Milnor $\bar{\mu}$-invariant of links and p is a prime (see [2]).

Then we can prove the following

THEOREM 1. (i) *$P_{n,q}$ is a normal subgroup of B_n and therefore of P_n.*

(ii) $[P_{n,q}, P_{n,r}] \subset P_{n,q+r}$ ([,] *denotes the commutator group*).

(iii) $\bigcap_q P_{n,q} = \{1\}$.