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On positive solutions of second order elliptic partial differential equations

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The paper studies necessary and sufficient conditions for the existence of positive solutions for the equation $-\Delta u+pu=0$ on a domain G in terms of the existence of a solution of a related Riccati inequality. Certain results from ordinary differential equations are extended to this setting providing sufficient conditions for positive solutions.

1. Introduction

The purposes of this paper are to extend known results on the existence of positive solutions, on disconjugacy and nonoscillation of the ordinary differential equation (o.d.e.)

$$u''(t) + p(t)u(t) = 0 \tag{1.1}$$

on a nontrivial interval I, which is possibly unbounded, to an analogous p.d.e.

$$-\Delta u(x, y) + p(x, y)u(x, y) = 0 \tag{1.2}$$

on a nontrivial domain, i.e., a connected open set in $\boldsymbol{R}^2$, or possibly on its closure. The coefficient function p in (1.1) is assumed to be continuous, while in (1.2) we assume a local Hölder continuity.

The results readily extend from equations (1.2) in $\boldsymbol{R}^2$ to those in $\boldsymbol{R}^n$, $n>2$; and where the second order Laplacian term $-\Delta u$ is replaced by a more general second term $-\nabla\cdot(r\nabla u)$, where $n>1$ and r is an $n\times n$ symmetric real matrix valued function which is in C^1, and which is uniformly positive definite.

The reason for the minus in (1.2), which is not present in (1.1), is the currently available literature in o.d.e.'s and in p.d.e.'s where these respective forms predominate.

The results to be presented in this paper are obtained by extending the following theorem of M. Bôcher [3] to p.d.e.'s.

THEOREM 1.1 (Bôcher). *Equation* (1.1) *has a positive solution on a non-*

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