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Some nonlinear degenerate diffusion equations with a nonlocally convective term in ecology

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1. Introduction

In the past several years, from an ecological point of view a number of authors (e.g. Gurney and Nisbet [11], Gurtin and MacCamy [12], Aronson [3], Newman [19] et al) have studied spatial spreading population models in which biological interactions and nonlinear diffusion process called "density-dependent dispersal" are taken into account. This nonlinear diffusion process is described by an equation of degenerate parabolic type.

In this paper, we are concerned with a model for the spatial diffusion of biological population which provides a kind of mechanism of aggregation and which is represented by equation

$$u_t = (u^m)_{xx} - \left[\left\{\int_{-\infty}^{\infty} K(x-y)u(y, t)dy\right\}u\right]_x, \ x \in \boldsymbol{R}^1, \ t > 0 \tag{1.1}$$

subject to an initial condition

$$u(x, 0) = u_0(x), \ x \in \boldsymbol{R}^1, \tag{1.2}$$

where $u(x, t)$ denotes the population density at point $x \in \boldsymbol{R}^1$ and at time $t>0$ and $1<m<\infty$. We assume the following assumptions on u_0 and K:

(A.1) $u_0 \geq 0$ on $\boldsymbol{R}^1$ and $u_0 \in L^1(\boldsymbol{R}^1) \cap L^\infty(\boldsymbol{R}^1)$;

(A.2) K is differentiable on $\boldsymbol{R}^1$ except for a finite number of discontinuity points of the first kind, $K \in L^\infty(\boldsymbol{R}^1)$ and $K' \in L^1(\boldsymbol{R}^1)$.

Here K' means dK/dx. In what follows we denote the problem (1.1), (1.2) by $P(K, u_0)$.

If the term containing K is absent, the equation (1.1) is reduced to the "porous media equation" occurring in the theory of flow through porous media (see. [5]). The most interesting phenomenon is that, because of the degeneracy of diffusion at $u=0$, an initial smooth disturbance with compact support spreads out at a finite speed (see. [20]) and loses the smoothness (see. [2] and [13]). This contrasts with the property of the heat conduction case ($m=1$). For the second term of the right hand side of (1.1), we give a specific function K defined by