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On the extendibility of vector bundles over the lens spaces and the projective spaces

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§1. Introduction

Let X and A be a topological space and its subspace. Then a fibre bundle ζ over A is said to be *extendible* to X, if there is a fibre bundle α over X whose restriction $\alpha|A$ to A is equivalent to ζ.

R. L. E. Schwarzenberger ([9; Appendix I], [21]) and several authors studied the extendibility of vector bundles over the complex (resp. real) projective n-space CP^n (resp. RP^n) to CP^m (resp. RP^m) for $m>n$ (cf., e.g., the references of [24]).

For an integer $q\geqq 2$, let L_q^n denote the standard lens space mod q or its n-skeleton:

$$L_q^{2i+1} = L^i(q) = S^{2i+1}/Z_q \text{ or } L_q^{2i} = \pi(S^{2i})(\pi: S^{2i+1} \longrightarrow L_q^{2i+1} \text{ is the projection}),$$

where $L_2^n = RP^n$. The purpose of this paper is to study the extendibility of complex (or real) vector bundles over L_q^n to L_q^m for $m>n$, as a continuation of the previous papers [18], [14] and [15].

Let η be the canonical complex line bundle over L_q^n, i.e., the induced bundle $\pi^*\eta$ of the one η over CP^i by the natural projection $\pi: L_q^{2i+1} \to CP^i$ or its restriction $\pi^*\eta\,|\,L_q^{2i}$. Then the main results on complex bundles are stated as follows:

THEOREM 1.1. *Let ζ be a complex t-plane bundle over L_q^n. Then ζ is stably equivalent to a complex $t'(=\sum_{i=1}^{q-1} b_i)$-plane bundle $\zeta'=\sum_{i=1}^{q-1} b_i\eta^i$ over L_q^n for some integers $b_i\geqq 0$. Furthermore, we have the following* (i) *and* (ii):

(i) *If $t\geqq [n/2]$, then ζ is extendible to L_q^{2t+1}. If $t\geqq[(n+1)/2]$ and $t\geqq t'$, then ζ is extendible to L_q^m for any $m\geqq n$.*

(ii) *Take a prime factor p of q with $p\leqq[n/2]+1$, and put $a=[n/2(p-1)]$ and*

$$c_k \equiv \textstyle\sum_l b_{lp+k} \bmod p^a,\ 0\leqq c_k < p^a, \qquad \textit{for} \quad 1\leqq k\leqq p-1.$$

If there is an integer m satisfying

$$t<m<p^a \ \textit{and} \ \textstyle\sum_{j_1+\cdots+j_{p-1}=m} \prod_{k=1}^{p-1} \binom{c_k}{j_k} k^{j_k} \not\equiv 0 \bmod p,$$