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# Numerical approximations to interface curves for a porous media equation

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## 1. Introduction

We are concerned with difference approximations to the initial value problem for the one dimensional porous media equation described by

$$v_t = (v^m)_{xx}, \quad t \in (0, +\infty), \quad x \in \boldsymbol{R}^1 \qquad (m > 1) \tag{1.1}$$

with an initial value

$$v(0, x) = v^0(x), \quad x \in \boldsymbol{R}^1, \tag{1.2}$$

where $v$ represents the density of an ideal gas flowing in a homogeneous porous medium which occupies all of $\boldsymbol{R}^1$. (1.1) is obtained by combining the equation of state, conservation of mass and Darcy's law ([8]). Physically, $v^{m-1}$ is the pressure of the gas and $(v^{m-1})_x$ is the velocity. From the reason that the diffusion rate of (1.1), $mv^{m-1}$ vanishes at points where $v=0$ because of $m>1$, (1.1) exhibits an interesting phenomenon of the finite speed of propagation of disturbances. In other words, when $v^0(x)$ has compact support, a solution $v(t, x)$ of (1.1), (1.2) has also compact support for any $t>0$. As an example to illustrate this property, we may show an explicit solution of (1.1) due to Barenblatt and Pattle ([3], [10]). This is of the form

$$v(t, x) = \begin{cases} \dfrac{1}{\lambda(t)}\left\{1-\left(\dfrac{x}{\lambda(t)}\right)^2\right\}^{1/(m-1)} & \text{for} \quad |x| \leqq \lambda(t),\ t \geqq 0, \\ 0 & \text{for} \quad |x| \geqq \lambda(t),\ t \geqq 0, \end{cases} \tag{1.3}$$

where

$$\lambda(t) = \left\{\frac{2m(m+1)}{m-1}(t+1)\right\}^{1/(m+1)} \qquad \text{for} \quad t \geqq 0 \tag{1.4}$$

(see Figs. 1 and 2).

We note here that the solution (1.3) is not a classical solution for $m \geqq 2$, in the sense that the trasition from the region of medium which contains gas ($v>0$) to the one which does not ($v=0$) is not smooth. The boundary, (1.4) in (1.3), is called the interface.