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Boundary value control of thermoelastic systems

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1. Introduction

Let Ω be a bounded domain in R^n with smooth boundary S, and consider an n-dimensional linear elastic solid occupying Ω in its non-deformed state. Let us denote by $u(x, t)=\{u_i(x, t)\}_{1\leqq i\leqq n}$ the displacement vector from $x=\{x_i\}_{1\leqq i\leqq n}$ at the time t of the material particle which lies at x in the non-deformed state. If the temperature of the medium is not taken into consideration, then $u_i(x, t)$ $(1\leqq i\leqq n)$ satisfy the system of equations

$$(1.1)\quad \rho(x)(\partial^2 u_i/\partial t^2)(x, t) = \textstyle\sum_{j=1}^n \partial\sigma_{ij}/\partial x_j + g_i(x, t) \quad \text{in} \quad \Omega\times(0, \infty),$$

where $\rho(x)$, $\sigma_{ij}(1\leqq i, j\leqq n)$ and $g(x, t)=\{g_i(x, t)\}_{1\leqq i\leqq n}$ denote the density, the stress tensors and the external force respectively. By Hook's law, there exists the linear dependence

$$\sigma_{ij} = \textstyle\sum_{k,l=1}^n a_{ijkl}\varepsilon_{kl}(u), \quad 1\leqq i, j\leqq n$$

between the stress tensors σ_{ij} and the linearized strain tensors

$$\varepsilon_{ij}(u) = (\partial u_i/\partial x_j + \partial u_j/\partial x_i)/2, \quad 1\leqq i, j\leqq n.$$

Here a_{ijkl} are, in general, functions in t and x, but independent of the strain tensors. The functions a_{ijkl} are called the coefficients of elasticity.

The problem of controlling the deformation of the medium by applying traction forces $f(x, t)=\{f_i(x, t)\}_{1\leqq i\leqq n}$ on the boundary as

$$(1.2)\qquad \textstyle\sum_{j=1}^n \nu_j(x)\sigma_{ij} = f_i(x, t) \quad \text{on} \quad S\times(0, \infty),$$

where $\nu(x)=\{\nu_i(x)\}_{1\leqq i\leqq n}$ is the outward unit normal vector at x on S, was considered by Clarke [1] and the author [15]. They obtained approximate controllability of the control system (1.1) with (1.2) when a_{ijkl} are independent of time t.

If the coefficients of elasticity are constants and further do not depend on the rotation of the coordinate axes, that is, if the elastic properties of the medium are the same in all directions, then the medium is said to be isotropic. In this case, a_{ijkl} are given by

$$(1.3)\qquad a_{ijkl} = \lambda\delta_{ij}\delta_{kl} + \mu(\delta_{il}\delta_{jk} + \delta_{ik}\delta_{jl}),$$