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Existence of oscillatory solutions for fourth order superlinear ordinary differential equations

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1. Introduction

In this paper we consider the fourth order Emden-Fowler type equation

$$y^{(4)} = p(t)|y|^{\alpha} \operatorname{sgn} y, \tag{1}$$

where $\alpha > 1$ is a constant and $p(t)$ is a positive continuous function on $[t_0, \infty)$, $t_0 > 0$. We are concerned with oscillatory and nonoscillatory properties of proper solutions of (1). A nontrivial real-valued solution $y(t)$ of (1) is called proper if it exists on some half-line $[T_y, \infty) \subset [t_0, \infty)$. A proper solution is called oscillatory if it has arbitrarily large zeros; otherwise it is called nonoscillatory.

We denote by $\mathscr{S}$ the set of all proper solutions of (1). From the viewpoint of oscillatory and nonoscillatory properties, $\mathscr{S}$ can be decomposed into a disjoint union

$$\mathscr{S} = \mathscr{O} \cup \mathscr{N},$$

where $\mathscr{O}$ (resp. $\mathscr{N}$) is the set of all oscillatory (resp. nonoscillatory) solutions of (1). Moreover $\mathscr{N}$ can be decomposed into a disjoint union

$$\mathscr{N} = \mathscr{N}_0 \cup \mathscr{N}_2 \cup \mathscr{N}_4,$$

where $\mathscr{N}_0$, $\mathscr{N}_2$ and $\mathscr{N}_4$ denote the sets of nonoscillatory solutions $y(t)$ satisfying,

$$y(t)y'(t) < 0,\ y(t)y''(t) > 0,\ y(t)y'''(t) < 0,$$

$$y(t)y'(t) > 0,\ y(t)y''(t) > 0,\ y(t)y'''(t) < 0$$

and

$$y(t)y'(t) > 0,\ y(t)y''(t) > 0,\ y(t)y'''(t) > 0$$

respectively, for all sufficiently large t. The following results are known:

THEOREM A (Kiguradze [3]). $\mathscr{N}_0 \neq \emptyset$.

THEOREM B (Kitamura [6]). *$\mathscr{N}_2 = \emptyset$ if and only if*

$$\int_{t_0}^{\infty} t^{2+\alpha} p(t)dt = \infty. \tag{2}$$