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Supplement to "Compact transformation groups on Z_2-cohomology spheres with orbit of codimension 1"

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§1. Introduction

In the main theorem of the previous paper [1], we have proved the following

(1.1) *Let (G, M) be a smooth action of a compact connected Lie group G on a connected closed smooth manifold M with orbit of codimension* 1. *If M is a Z_2-cohomology sphere, then (G, M) is (essentially) isomorphic to*

(a) *the linear action on the sphere S^n via a representation $G \to SO(n+1)$,*

(b) *the standard action on the Brieskorn manifold $W^{2m-1}(r)$ for odd $r \geqq 1$, given in* [1; Ex. 1.2], *or*

(c) *the action $(SO(4), M)$ with* $\dim M = 7$, *given in* [1; Ex. 1.3], *which exists for each relatively prime integers l_s and m_s $(s=1, 2)$ with*

$$l_s \equiv m_s \equiv 1 \bmod 4, \quad 0 < l_1 - m_1 \equiv 4 \bmod 8, \quad l_2 - m_2 \equiv 0 \bmod 8.$$

The purpose of this supplement is to prove the following (1.2) whose sufficiency is asserted in [1; Ex. 1.3]:

(1.2) *Among the actions $(SO(4), M)$ in* (c) *of* (1.1), *M is a homotopy sphere if and only if $(l_1, m_1, l_2, m_2) = (1, -3, 1, 1)$, and then $M = S^7$ and the action is linear.*

By virtue of (1.2), the following theorem is an immediate consequence of (1.1), because it is well-known that $W^{2m-1}(r)$ in (b) is a homotopy sphere if and only if both m and r are odd (cf. [2; Satz 1]).

THEOREM 1.3. *If M is a homotopy sphere in addition, then (G, M) in* (1.1) *is (essentially) isomorphic to a linear action in* (a) *or the action on $W^{2m-1}(r)$ in* (b) *for odd m and odd $r \geqq 1$.*

We prepare some lemmas on the cohomology of certain coset spaces of $S^3 \times S^3$ in § 2, and prove (1.2) in § 3.