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On the relatively smooth subhyperalgebras of hyperalgebras

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Introduction

Let H be a hyperalgebra (i.e., a cocommutative irreducible Hopf algebra) over a field k. Then for any cocommutative coalgebra C over k the set $\mathrm{Hom}_{coal}(C, H)$ of all coalgebra homomorphisms of C into H has a group structure. If $\rho: H \to J$ is a homomorphism of hyperalgebras, then ρ induces a group homomorphism of $\mathrm{Hom}_{coal}(C, H)$ into $\mathrm{Hom}_{coal}(C, J)$. It is known that if

$$(*) \qquad k \longrightarrow G \longrightarrow H \longrightarrow J \longrightarrow k$$

is an exact sequence of hyperalgebras, then the induced sequence

$$e \longrightarrow \mathrm{Hom}_{coal}(C, G) \longrightarrow \mathrm{Hom}_{coal}(C, H) \longrightarrow \mathrm{Hom}_{coal}(C, J)$$

of groups is exact for any cocommutative coalgebra C ([13, Proposition 14.12]).

In [15] Yanagihara showed that if the exact sequence $(*)$ is split, then the induced sequence

$$(**) \quad e \longrightarrow \mathrm{Hom}_{coal}(C, G) \longrightarrow \mathrm{Hom}_{coal}(C, H) \longrightarrow \mathrm{Hom}_{coal}(C, J) \longrightarrow e$$

is exact and split for any C. On the other hand, it is proved by Dieudonné in [1, Proposition 8, Chapter 2] that when H and J in $(*)$ are of finite type and reduced over a perfect field k, the sequence $(**)$ is exact for any C if and only if G is reduced. Moreover Takeuchi showed in [10, Theorem 1.8.1] that if the homomorphism $H \to J$ is smooth in the sense of [10], then $(**)$ is exact for any C. (Actually, the smoothness of the homomorphism is stronger than the exactness of $(**)$.)

In this paper we will generalize the above results and give several characterizations for $(**)$ to be exact.

Let G be a subhyperalgebra of a hyperalgebra H and $J = H/HG^+$ the quotient coalgebra. Consider the sequence

$$k \longrightarrow G \longrightarrow H \longrightarrow J \longrightarrow k,$$

where $G \to H$ is the inclusion and $H \to J$ is the natural projection. We will prove in Section 1 that the following conditions are equivalent: (1) The induced map $\mathrm{Hom}_{coal}(C, H) \to \mathrm{Hom}_{coal}(C, J)$ is surjective for any cocommutative coalgebra